Patterns and rules

Find the rule behind a pattern, then use it.

Continue number sequences, grow shape patterns from sticks, run number machines forwards and backwards, find missing numbers, test odd and even rules, meet square and triangle numbers, and jump far ahead in a pattern.

YOU ARE READY FOR THIS BOOK IF

You can continue the pattern 3, 6, 9, 12 for three more numbers.

One step up	M3.8 Letters for numbers and simple equations
This book	**M2.8 Patterns and rules**
One step down	M1.8 Patterns

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M2.8.6 Square and triangle numbers

M2.8.7 Jumping far ahead

Key words, A to Z

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M2.8.1 CHAPTER 1

Number sequences

What number comes next in 3, 7, 11, 15?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** It goes up by 4 each time, so 19.
- ☐ **B** It goes up by 3 each time, so 18.
- ☐ **C** It doubles each time, so 30.

BY THE END YOU CAN

- Continue a sequence that goes up or down in equal steps
- Find the step and the missing numbers in a sequence

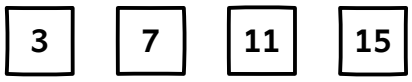
Look at this list of numbers: 3, 7, 11, 15. What comes next?

A **sequence** is a list of numbers. The numbers follow a rule.

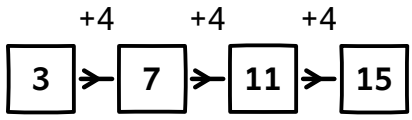
Here, each number is 4 more than the one before. The jump of 4 is the **step**.

DRAW IT FIRST

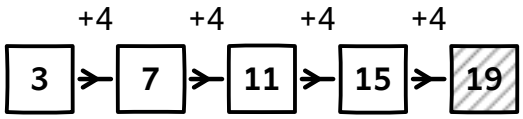
Find the step, then the next number



1 Write the numbers in boxes: 3, 7, 11, 15.



2 Look at each jump. 3 to 7 is +4. 7 to 11 is +4. 11 to 15 is +4.

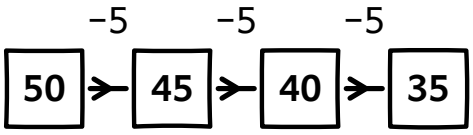


3 Jump 4 more. 15 + 4 = 19. The next number is 19.

To find the step, take the first number from the second. 7 − 3 = 4.

Then check every pair. 11 − 7 = 4 and 15 − 11 = 4. The step is the same each time.

A sequence can go down too. This one goes down by 5 each time.



Each number is 5 less than the one before. 50 − 45 = 5.

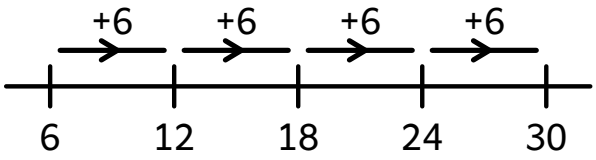
WORKED EXAMPLE

Continue 2, 5, 8, 11 with the next two numbers.

1. Find the step. 5 − 2 = 3.
2. Check the other pairs. 8 − 5 = 3 and 11 − 8 = 3.
3. Add 3 to the last number. 11 + 3 = 14.
4. Add 3 again. 14 + 3 = 17.

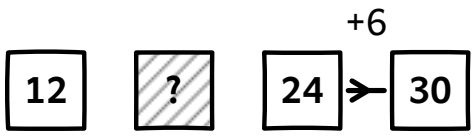
Answer: 14, 17

You can show a sequence on a number line. Each hop is one step.



The hops are all the same length. The step is 6.

Sometimes a number is missing. The last two numbers tell you the step.



Only the last jump is known. It is +6.

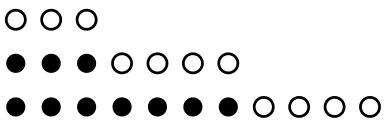
WORKED EXAMPLE

Find the missing number in 12, ?, 24, 30.

1. Use the last two numbers. 30 − 24 = 6. The step is 6.
2. The missing number is one step after 12. 12 + 6 = 18.
3. Check. 18 + 6 = 24. It fits.

Answer: 18

SEE IT THREE WAYS

With things	As a picture	In symbols
Make a pile of 3 stones. Make a pile of 7. Make a pile of 11. Each pile is 4 stones bigger.		**3, 7, 11, 15. The step is 4. 7 − 3 = 4. 11 − 7 = 4. 15 − 11 = 4.**

TRY IT

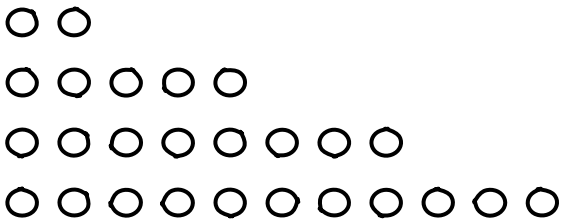
Grow a sequence with stones

You need: about 26 stones, beans or seeds, a pencil and paper.

1. Put 2 stones in a row. Write 2.
2. Make a new row with 3 more stones than the first. It has 5. Write 5.
3. Add 3 more again. Write the new number.
4. Do it once more. Write the number.

What you will see: Each row is 3 stones longer than the row above. Your numbers are 2, 5, 8, 11. The step is 3 every time.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~The numbers 4, 8, 12 double each time. The next number is 24.~~

Why it feels right: 4 to 8 is a double. So it looks like doubling.

The test that breaks it: Check the next pair. Double 8 is 16, but the list says 12. The rule fails on the second pair.

What is true: Check the step on every pair. 8 − 4 = 4 and 12 − 8 = 4. The rule is add 4, so the next number is 16.

KEY WORDS

sequence
a list of numbers that follows a rule.

step
how much the numbers go up or down each time.

BACK TO YOUR GUESS

The answer is A: It goes up by 4 each time, so 19.

The numbers go up by 4 each time. 15 + 4 = 19. The list is a sequence. The jump is its step.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. What comes next? 6, 11, 16, 21, …

   A 26

   B 22

   C 42

Answer at the back of the book.

2. Write the next two numbers: 90, 80, 70, 60, ...

   Answer at the back of the book.

3. Find the missing number: 8, 14, ?, 26, 32

   Answer at the back of the book.

4. Mei says 2, 4, 7 goes up by 2 each time. Is she right?

   A No. The first jump is 2 and the second is 3.

   B Yes. 2 to 4 is up by 2.

   Answer at the back of the book.

5. Write the first five numbers of a sequence. It starts at 100 and goes down by 20 each time.

   Answer at the back of the book.

6. What comes next? 47, 41, 35, 29, ...



   A 35

   B 17

   C 23

   Answer at the back of the book.

7. Ravi writes 5, 9, 13, 17. What number comes next?

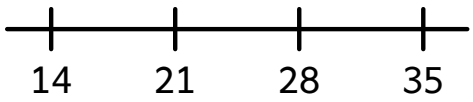
   A 22

   B 21

   C 34

   Answer at the back of the book.

8. What is the step in 14, 21, 28, 35?



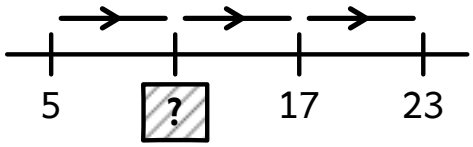
A 21

B 14

C 7

Answer at the back of the book.

9. Hana marks 5, a gap, 17 and 23 in equal hops. What number fills the gap?



A 11

B 10

C 12

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

A sequence is a list of numbers that follows a rule. Its step is the same each time. For 3, 7, 11 the step is 7 − 3 = 4. Check every pair.

Draw: Four boxes with 3, 7, 11 and 15. Write +4 on each jump between them.

M2.8.2 CHAPTER 2

Growing shape patterns

How many sticks do I need for a row of 5 squares?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** 20 sticks. Each square needs 4 sticks.
- ☐ **B** 16 sticks. The first square needs 4, and each new one adds 3.
- ☐ **C** 15 sticks. Each square needs 3 sticks.

BY THE END YOU CAN

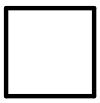
- Draw the next shape of a growing pattern of squares
- Make a table of the number of squares and sticks

You can make a row of squares from sticks. Let us count the sticks.

Each new shape has one more square than the shape before. This is a **growing pattern**.

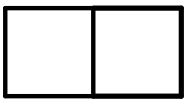
DRAW IT FIRST

Grow *a* row of squares



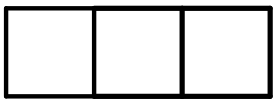
4 sticks

1 Make one square. It needs 4 sticks.



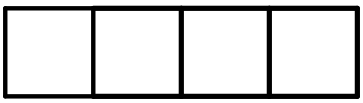
4 + 3 = 7 sticks

2 Add *a* square *a*t the end. It shares *a* stick, so *a*dd only 3. 4 + 3 = 7.



7 + 3 = 10 sticks

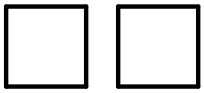
3 Add another square. 7 + 3 = 10.



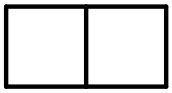
10 + 3 = 13 sticks

4 Add one more. 10 + 3 = 13.

A new square uses the end stick of the last square. That stick is counted once.



*a*part: 8 sticks



joined: 7 sticks

Squares that touch share *a* stick. Squares *a*part do not.

A ***table*** puts the squares *a*nd the sticks side by side.

Squares	Sticks
1	4
2	7
3	10
4	13
5	16

The sticks go up by 3 each row. The shaded box answers our question.

WORKED EXAMPLE

How many sticks make a row of 5 squares?

1. Fill a table. 1 square needs 4 sticks.
2. Add 3 for each new square. 4, 7, 10, 13, 16.
3. 5 squares need 16 sticks.

Answer: 16 sticks

There is a quick way too. Take 3 times the squares. Then add 1.

Why? Each square has 3 sticks of its own. One more stick closes the first square.

For 5 squares: 3 × 5 + 1 = 16. It gives the same answer as the table.

SEE IT THREE WAYS

With things

Use sticks. Make 1 square. Then join a second square, then a third. Count the sticks each time.

As a picture



In symbols

4, 7, 10. The step is 3. 7 − 4 = 3. 10 − 7 = 3.

TRY IT

Build squares with sticks

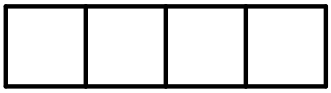
You need: about 16 sticks of the same length, such as twigs or straws, a pencil and paper.

Be safe: Ask a grown up to cut the sticks. Never use matches.

1. Make one square with 4 sticks. Write 4.
2. Add a square at the end. Use the end stick of the first square.
3. Count the new sticks you used. Write the total.
4. Keep adding squares up to 4. Write each total.

What you will see: Each new square needs only 3 sticks. Your totals are 4, 7, 10, 13.

If you cannot try it, this is what happens.



4 squares, 13 sticks

A WRONG IDEA MANY PEOPLE HAVE

~~**Two squares in a row need 8 sticks. Each square needs 4.**~~

Why it feels right: One square has 4 sticks. So two squares seem to need 4 + 4.

The test that breaks it: Build two squares that touch, with real sticks. Count them. You use 7. The middle stick is a side of both squares.

What is true: Joined squares share sticks. Each new square adds 3, not 4. Two squares need 7 sticks.

KEY WORDS

growing pattern
a pattern where each shape is bigger than the shape before.

table
numbers in rows and columns, such as squares and sticks.

BACK TO YOUR GUESS

The answer is B: 16 sticks. The first square needs 4, and each new one adds 3.

Squares in a row share sticks. The first square needs 4. Each new square adds 3. So 5 squares need 16 sticks.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. How many sticks make a row of 6 squares?

   Answer at the back of the book.

2. Kofi says 3 squares in a row need 12 sticks. Is he right?

   A No. They need 10 sticks.

   B Yes. 3 × 4 = 12.

   C No. They need 9 sticks.

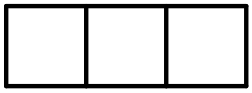
   Answer at the back of the book.

3. Fill the empty boxes. Find the sticks for 3, 4 and 5 squares.

Squares	Sticks
1	4
2	7
3	
4	
5	

   Answer at the back of the book.

4. A pattern grows from 1 square, to 2, to 3. Here is the shape with 3. Draw the next shape. How many sticks does it need?



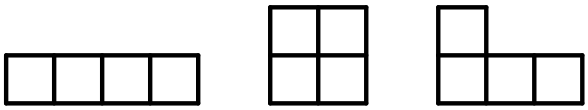
Answer at the back of the book.

5. **From an earlier chapter**

Find the missing number: 5, 9, ?, 17, 21

Answer at the back of the book.

6. A row grows 1 square, 2 squares, 3 squares. Which shape comes next?



A Shape A

B Shape B

C Shape C

Answer at the back of the book.

7. Amina made this table. One row is wrong. Which row?

Squares	Sticks
1	4
2	7
3	10
4	12
5	16

A 5 squares: 16

B 3 squares: 10

C 4 squares: 12

Answer at the back of the book.

8. Sipho joins 10 squares in a row. How many sticks does he need?

A 30

B 40

C 34

D 31

Answer at the back of the book.

9. Ana used 19 sticks to make a row of joined squares. How many squares is that?

A 6

B 5

C 4

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Squares in a row share sticks. One square needs 4 sticks. Each new square adds 3. So 2 squares need 7, 3 need 10, and 4 need 13.

Draw: A row of 3 squares made from sticks. Beside it, a table of squares and sticks.

M2.8.3 CHAPTER 3

Number machines

A machine does × 3, then + 1. What comes out if I put in 4?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** 15. Add 1 first, then times 3.
- ☐ **B** 13. Times 3 gives 12, then add 1.
- ☐ **C** 12. The machine only times by 3.

BY THE END YOU CAN

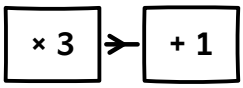
- Find the output of a machine like 'times 3, then add 1'
- Find the rule from pairs of inputs and outputs
- Work backwards from an output to find the input

A **number machine** changes numbers by a rule. Our machine does two jobs, in order.

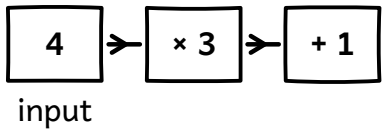
The number you put in is the **input**. The number that comes out is the **output**.

DRAW IT FIRST

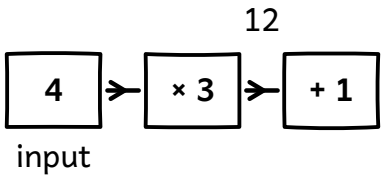
Run *a* number through the *m*achine



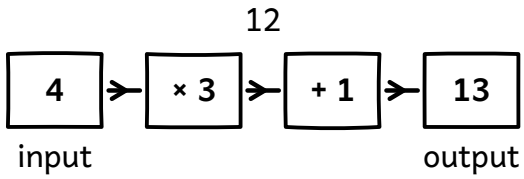
1 The machine does two jobs. First × 3. Then + 1.



2 Put in 4. This is the input.

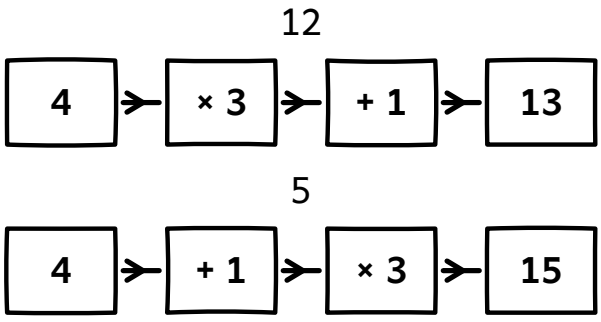


3 The first job gives 4 × 3 = 12.



4 The second job gives 12 + 1 = 13. This is the output.

The order matters. Now do + 1 first, then × 3. Put in 4 *a*gain.



The *s*ame jobs in *a* different order give *a* different output.

WORKED EXAMPLE

Put 7 in the machine. It does × 3, then + 1.

1. First job: 7 × 3 = 21.
2. Second job: 21 + 1 = 22.
3. The output is 22.

Answer: 22

Squares of sticks use this machine too. Put in 4 squares. 13 sticks come out.

It is the stick rule from the last chapter.

Sometimes you see the inputs and the outputs, but not the rule.

Input	Output
1	5
2	7
3	9

What does the machine do?

WORKED EXAMPLE

Find the rule. 1 gives 5, 2 gives 7, 3 gives 9.

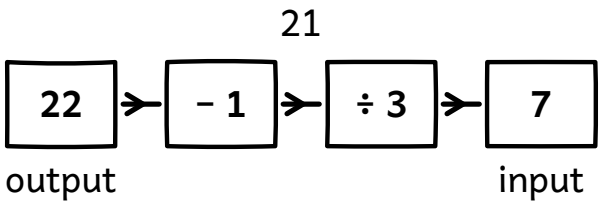
1. The inputs go up by 1. The outputs go up by 2. So the rule starts with × 2.
2. Try × 2 on the input 1. 2 × 1 = 2. The output is 5.
3. It needs 3 more. 5 − 2 = 3. So add 3.
4. Test every pair. 2 × 2 + 3 = 7. 2 × 3 + 3 = 9.

Answer: × 2, then + 3

Now go backwards. The output is 22. What did you put in?

Undo each job with its **opposite**. Start with the last job.

The opposite of + is −. The opposite of × is ÷.



Start at the output. Undo the last job first.

WORKED EXAMPLE

The machine does × 3, then + 1. The output is 22. Find the input.

1. The last job was + 1. Undo it first. Take 1 away. 22 − 1 = 21.
2. The first job was × 3. Undo it. 21 ÷ 3 = 7.
3. Check. 7 × 3 + 1 = 22.

Answer: 7

SEE IT THREE WAYS

With things	**As a picture**	**In symbols**
Put in 2. Make 2 groups of 3 beans. Then add 1 more bean. Count them all.		**2 × 3 + 1 = 7. The input is 2. The output is 7.**

TRY IT

Be the number machine

You need: a pencil and paper, a friend, if you have one.

1. Write a rule: × 2, then + 1. Keep it hidden from your friend.
2. Ask your friend for the inputs 1, 2, 3 and 4. Give each output.
3. Write the inputs and outputs in a table.
4. Ask your friend to find the rule. On your own, cover the rule and find it.

What you will see: The outputs 3, 5, 7, 9 go up by 2. That shows the × 2. The first output needs 1 more.

If you cannot try it, this is what happens.

Input	Output
1	3
2	5
3	7
4	9

A WRONG IDEA MANY PEOPLE HAVE

~~To go backwards from 22, divide by 3 first. Then take 1 away.~~

Why it feels right: The machine does × 3 first. So it seems you undo × 3 first.

The test that breaks it: 22 ÷ 3 leaves a bit over. Say you use 7, then take 1 away to get 6. Put 6 in the machine. 6 × 3 + 1 = 19, not 22.

What is true: Undo the last job first. You tie string round a parcel last. To open it, you untie the string first.

KEY WORDS

number machine
a rule that changes a number, one job at a time.

input
the number you put in.

output
the number that comes out.

opposite
the job that undoes another. + and − are opposites.

BACK TO YOUR GUESS

The answer is B: 13. Times 3 gives 12, then add 1.

The machine does its jobs in order. 4 × 3 = 12, then 12 + 1 = 13. Add 1 first and you get 15, which is a different machine.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. A machine does × 5, then − 2. Put in 6. What comes out?

   Answer at the back of the book.

2. A machine does + 2, then × 4. Put in 3. What comes out?

   A 20

   B 14

   C 9

   Answer at the back of the book.

3. Find the rule. 2 gives 9, 3 gives 13, 4 gives 17.

Input	Output
2	9
3	13
4	17

   Answer at the back of the book.

4. A machine does × 2, then + 5. The output is 19. What went in?

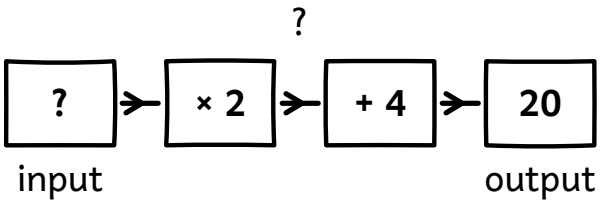
   Answer at the back of the book.

5. **From an earlier chapter**

   How many sticks make a row of 7 squares?

   Answer at the back of the book.

6. A machine does × 2, then + 4. The output is 20. What went in?



   A 6

   B 8

   C 48

   Answer at the back of the book.

7. Find the rule. 1 gives 5, 2 gives 8, 3 gives 11.

Input	Output
1	5
2	8
3	11

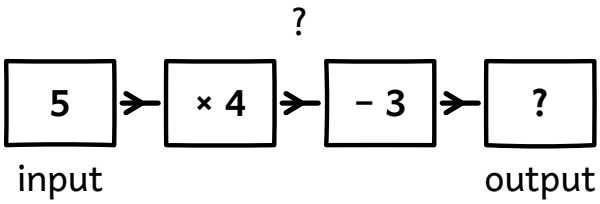
   A + 3

   B × 5

   C × 3, then + 2

   Answer at the back of the book.

8. A machine does × 4, then − 3. Put in 5. What comes out?



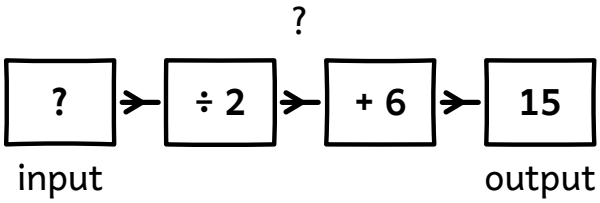
A 23

B 17

C 8

Answer at the back of the book.

9. Leila's machine does ÷ 2, then + 6. The output is 15. What went in?



A 24

B 9

C 18

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

A number machine does jobs in order. The input goes in. The output comes out. To go backwards, undo the last job first, with its opposite.

Draw: Four boxes in a row: input 4, × 3, + 1, output 13. Write 12 above the middle arrow.

M2.8.4 CHAPTER 4

Missing numbers

What is the missing number in 36 + ? = 50?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** 86. Add 36 and 50.
- ☐ **B** 14. Take 36 away from 50.
- ☐ **C** 26. Take the smaller digit from the bigger in each place.

BY THE END YOU CAN

- Find the missing number in 36 + ? = 50
- Find the missing number in ? × 6 = 42
- Use the opposite operation to check a missing number

Here is a sum with a hidden number: 36 + ? = 50.

The ? stands for a **missing number**. Think: 36 plus what makes 50?

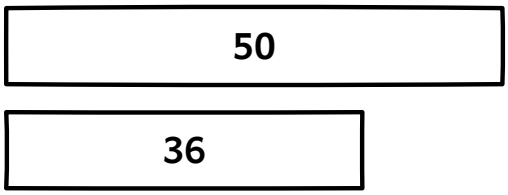
In the pictures, a hatched box with a ? marks it.

DRAW IT FIRST

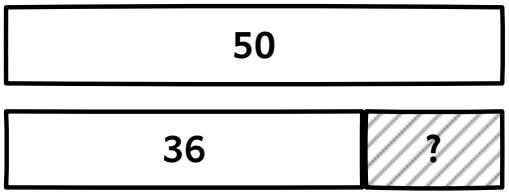
See the missing part



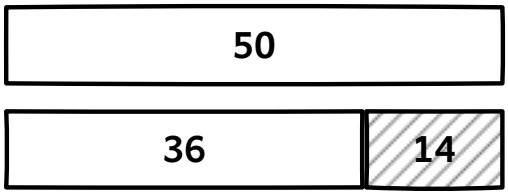
1 Draw the whole, 50, as one long box.



2 Under it, draw the part you know. It is 36.



3 The rest of the box is the missing part. Mark it ?.



50 − 36 = 14

4 Take the part you know from the whole. 50 − 36 = 14.

The whole is 50. Take away the part you know. That leaves the missing part.

WORKED EXAMPLE

Find the missing number. 36 + ? = 50.

1. The whole is 50. The part you know is 36.
2. Take the known part away. 50 − 36 = 14.
3. Check. Put 14 where the ? is. 36 + 14 = 50.

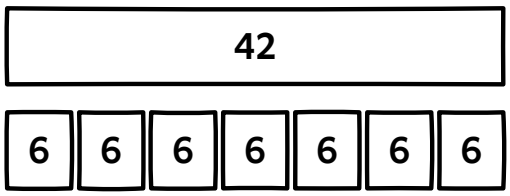
Answer: 14

Times works the same way. Look at ? × 6 = 42. What times 6 makes 42?

Use the **opposite operation**. It is the job that undoes another job.

Job	Opposite
+	−
−	+
×	÷
÷	×

Each job and the job that undoes it.



How many sixes fit in 42? Divide 42 by 6.

WORKED EXAMPLE

Find the missing number in ? × 6 = 42.

1. The job is × 6. Its opposite is ÷ 6.
2. Divide 42 by 6. 42 ÷ 6 = 7.
3. Check. Put 7 where the ? is. 7 × 6 = 42.

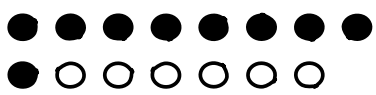
Answer: 7

SEE IT THREE WAYS

With things

Put out 9 stones. You need 15 in all. Add stones until you have 15. You add 6.

As a picture



In symbols

9 + ? = 15. The opposite of + is −. 15 − 9 = 6.

TRY IT

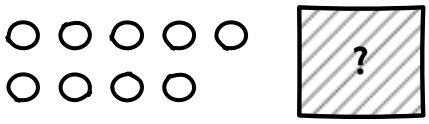
Find the hidden stones

You need: 15 stones, beans or seeds, *a* cup or *a* bowl, *a* pencil and paper.

1. Count out 15 stones. Write 15.
2. Cover some stones with the cup. Do not count them.
3. Count the stones you can see. Write that number, then + ? = 15.
4. Take the number you can see away from 15. That is your answer.
5. Lift the cup and count the hidden stones.

What you will see: Your answer matches the hidden stones. Take away finds the missing number in *a* sum with +.

If you cannot try it, this is what happens.



9 + ? = 15

A WRONG IDEA MANY PEOPLE HAVE

~~**In 36 + ? = 50, you see *a* +. So the ? is 36 + 50 = 86.**~~

Why it feels right: You see *a* plus sign, so your head says add.

The test that breaks it: Put 86 where the ? is. 36 + 86 = 122, not 50. The missing number must be less than 50.

What is true: The total, 50, is on the other side of the =. Take the part you know away: 50 − 36 = 14.

KEY WORDS

missing number
a hidden number, which you find.

opposite operation

the job that undoes another job.

BACK TO YOUR GUESS

The answer is B: 14. Take 36 away from 50.

36 + 14 = 50. The whole is 50 and you know one part, 36. Take the part away from the whole. 50 − 36 = 14.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Find the missing number. 29 + ? = 71

   Answer at the back of the book.

2. Find the missing number in ? × 8 = 56.

   Answer at the back of the book.

3. Amina says the ? in 25 + ? = 60 is 85. Is she right?

   A No. 25 + 85 = 110. The missing number is 35.

   B Yes. 25 + 60 = 85.

   C No. The missing number is 45.

   Answer at the back of the book.

4. Tariq has 48 beans. He gets some more and has 72. How many more did he get?

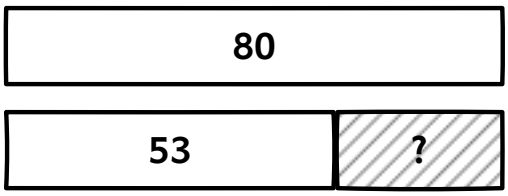
   Answer at the back of the book.

5. **From an earlier chapter**

   A machine does × 4, then + 3. The output is 31. What went in?

   Answer at the back of the book.

6. Kofi finds the missing part. What is the hatched part?



A 33

B 133

C 27

Answer at the back of the book.

7. Find the missing number in ? × 9 = 63.

A 54

B 7

C 8

Answer at the back of the book.

8. Mei finds that 50 − 36 = 14. Which sum checks her answer?

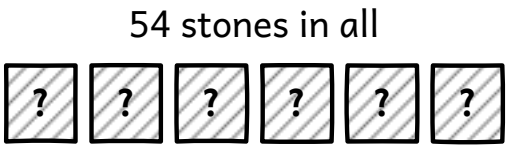
A 50 + 14 = 64

B 36 + 14 = 50

C 36 − 14 = 22

Answer at the back of the book.

9. Sipho puts 54 stones in 6 equal bags. How many stones are in each bag?



A 9

B 48

C 8

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

To find *a* missing number, use the opposite operation. For 36 + ? = 50, do 50 − 36 = 14. For ? × 6 = 42, do 42 ÷ 6 = 7. Then check.

Draw: A long box 50. Under it, *a* box 36 and *a* hatched box with *a* ?.

M2.8.5 CHAPTER 5

Odd and even rules

Is 7 + 9 odd or even? Do I have to add them to know?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Odd, because 7 and 9 are both odd.
- ☐ **B** Even, because the two left over ones make a pair.
- ☐ **C** I cannot tell. I have to add them first.

BY THE END YOU CAN

- Explain why odd + odd always makes an even number
- Test a rule with many examples before you trust it

Take 6 stones. Put them in pairs. You get 3 pairs. Nothing is left over.

Now take 7 stones. You get 3 pairs and 1 left over.

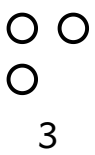
An **even** number makes pairs with none left over. An **odd** number has one left over.

Look at the last digit. A last digit of 0, 2, 4, 6 or 8 means even. Any other last digit means odd.

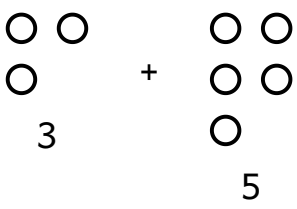
What happens when you add two odd numbers? Watch 3 + 5.

DRAW IT FIRST

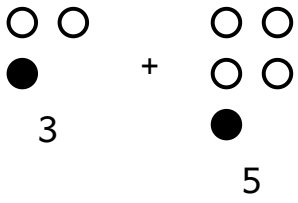
Add two odd numbers in pairs



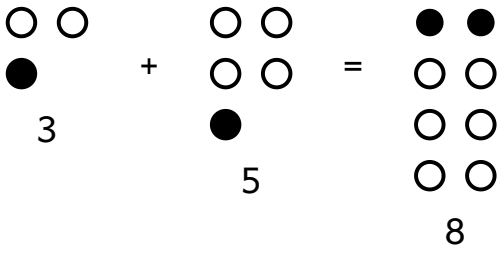
1 Put 3 dots in pairs. One pair, and one dot has no partner.



2 Put 5 dots in pairs. Two pairs, and one dot has no partner.



3 Fill in the two dots with no partner. They are the left overs.



4 Put all the dots together. The two left overs make a pair. 8 dots, none left over.

Each odd number has one left over. The two left overs make a pair. So nothing is left over, and the total is even.

Add	Total is
even + even	even
odd + even	odd
odd + odd	even

The three sums. Odd + odd is the one that surprises people.

Odd + odd = even is a **rule**. A rule tells you what happens every time.

How do you know a rule is right? Test it on many pairs. Use small numbers and big numbers.

Odd + odd	Total	Even
3 + 5	8	yes
7 + 9	16	yes
11 + 15	26	yes
21 + 33	54	yes

Four tests. Each total ends in 0, 2, 4, 6 or 8.

Many examples make you trust a rule. But you cannot test every pair of numbers. So ask why it works. The pairs picture tells you why.

One example that fails is enough to break a rule.

WORKED EXAMPLE

Is 17 + 25 odd or even? Do not add.

1. 17 ends in 7, so 17 is odd.
2. 25 ends in 5, so 25 is odd.
3. Odd + odd is even. So 17 + 25 is even.
4. Check: 17 + 25 = 42. 42 ends in 2, so it is even.

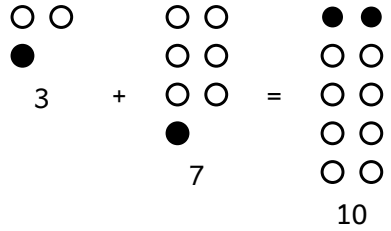
Answer: even

SEE IT THREE WAYS

With things

Put 3 stones in pairs, and 7 stones in pairs. Each pile has one left over. Push the piles together. The left overs make a pair.

As a picture



In symbols

3 + 7 = 10. Odd + odd = even.

TRY IT

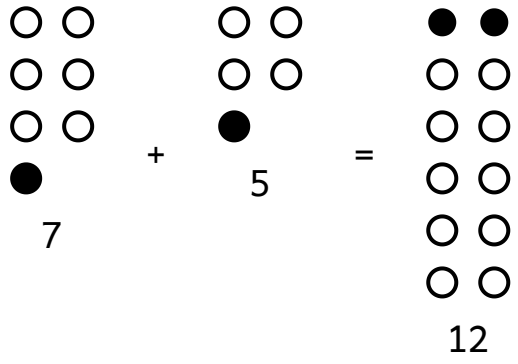
Test the rule with stones

You need: about 25 stones, beans or bottle tops, a pencil and paper.

1. Count out 7 stones. Put them in pairs. One is left over.
2. Count out 5 stones in a second pile. Put them in pairs. One is left over.
3. Push the two piles together. Pair up all the stones.
4. Write down if any stone is left over. Then try 9 and 3. Then try 11 and 1.

What you will see: The two left overs find each other every time. No stone is left over, so each total is even.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~Three odd numbers add up to an even number.~~

Why it feels right: The rule worked for two odd numbers. It feels like it should work for three.

The test that breaks it: Add 1 + 3 + 5. You get 9, and 9 is odd. One example fails, so the rule is broken.

What is true: Three odd numbers make an odd total. Two left overs make a pair. The third one stays alone.

KEY WORDS

even
a number that makes pairs with none left over.

odd
a number that has one left over when you make pairs.

rule
something that is true every time.

BACK TO YOUR GUESS

The answer is B: Even, because the two left over ones make *a* pair.

Odd + odd is even. Each odd number has one left over. The two left overs make *a* pair. So 7 + 9 is even. And 7 + 9 = 16.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Which sum gives an odd number?

   A 6 + 8

   B 7 + 9

   C 7 + 8

   Answer at the back of the book.

2. Without adding, say if 13 + 27 is odd or even.

   Answer at the back of the book.

3. Is 9 + 6 odd or even? Draw 9 dots and 6 dots in pairs to see.

   Answer at the back of the book.

4. Is 5 + 7 + 9 odd or even? Add the first two, then add 9.

   Answer at the back of the book.

5. Tariq tests "even + even = even". He tries 2 + 4, 6 + 8 and 10 + 12. It works each time. What should he do next?

   A Trust it now. Three examples prove it.

   B Test more pairs, and ask why it works.

   C Stop. A rule can never be trusted.

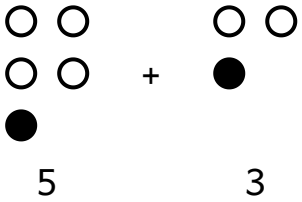
   Answer at the back of the book.

6. **From an earlier chapter**

   Find the missing number: 29 + ? = 61.

   Answer at the back of the book.

7. Mei puts 5 dots and 3 dots in pairs. Each has one left over dot. What happens when she joins them?



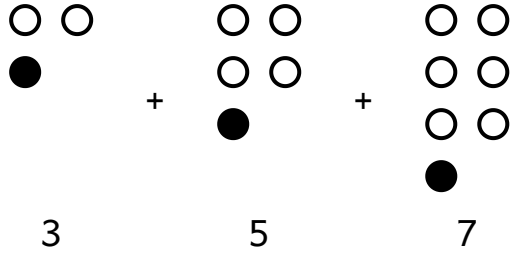
   A The two left overs stay alone. The total is odd.

   B The two left overs make a pair. The total is even.

   C 5 is odd, so the total is odd.

   Answer at the back of the book.

8. Sipho adds 3 + 5 + 7. Is the total odd or even?



   A Odd. Two left overs make a pair, but the third stays alone.

   B Even. Odd + odd is even.

   C I cannot tell without adding.

   Answer at the back of the book.

9. Kofi says two numbers always add to an even number. He tests 2 + 4 and 3 + 5. Both work. Is his rule true?

   A No. 3 + 5 = 8 breaks it.

   B Yes. Both tests worked.

   C No. 3 + 4 = 7 breaks it.

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Even numbers make pairs with none left over. Odd numbers have one left over. Odd + odd is even, because the two left overs make a pair. One example that fails breaks a rule.

Draw: Draw 3 dots and 5 dots in pairs. Fill in the left overs. Then draw all 8 dots in pairs.

M2.8.6 CHAPTER 6

Square and triangle numbers

Why are 4 and 9 called square numbers? Are they really squares?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** It is only a name. Squares have nothing to do with it.
- ☐ **B** That many dots make a square, with as many rows as columns.
- ☐ **C** Square numbers go up by the same step each time.

BY THE END YOU CAN

- Draw dots to show the square numbers 1, 4, 9 and 16
- Find the next triangle numbers after 1, 3, 6 and 10

Put 4 stones in 2 rows of 2. They make a square. Put 9 stones in 3 rows of 3. That is a square too.

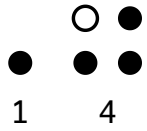
A **square number** is the number of dots in a square. There are as many rows as columns.

DRAW IT FIRST

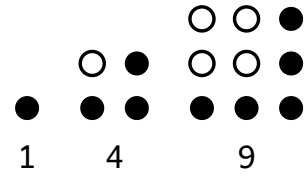
Build squares of dots



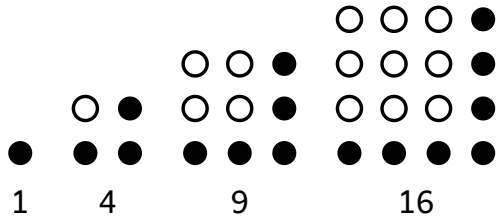
1 One dot is a square with 1 row of 1. It is 1 × 1 = 1.



2 Add a row and a column. You get 2 rows of 2. It is 2 × 2 = 4.



3 Add another row and column. You get 3 rows of 3. It is 3 × 3 = 9.



4 Once more. You get 4 rows of 4. It is 4 × 4 = 16.

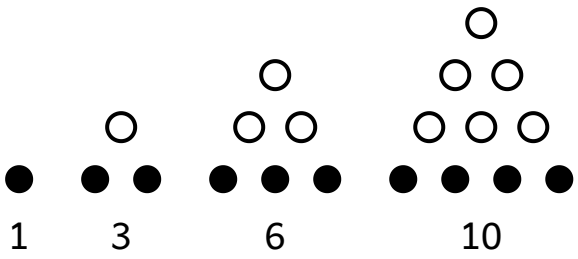
Square	Total	Added
1 × 1	1	1
2 × 2	4	3
3 × 3	9	5
4 × 4	16	7
5 × 5	25	9

The Added column counts the filled dots in the picture.

Look at the filled dots. The new dots are 3, then 5, then 7, then 9. They are the odd numbers.

Now try a different shape. Put 1 stone. Below it put a row of 2. Then a row of 3.

Each new row has one more dot. You make a triangle. A **triangle number** is the number of dots in a triangle.



The new row of each triangle is filled in.

From 1 to 3 you add 2. From 3 to 6 you add 3. From 6 to 10 you add 4. The row you add grows by 1 each time.

Rows	New row	Total
1	1	1
2	2	3
3	3	6
4	4	10
5	5	15
6	6	21
7	7	28

Add the new row to the total above it.

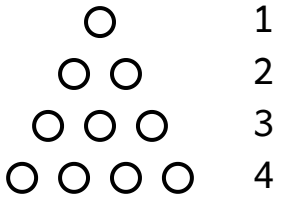
WORKED EXAMPLE

Find the next three triangle numbers after 10.

1. The next row is the 5th row. It has 5 dots. 10 + 5 = 15.
2. The 6th row has 6 dots. 15 + 6 = 21.
3. The 7th row has 7 dots. 21 + 7 = 28.

Answer: 15, 21, 28

SEE IT THREE WAYS

With things	As a picture	In symbols
Put 1 stone in the top row. Then 2, then 3, then 4. Count all the stones.	 1 2 3 4 1 + 2 + 3 + 4 = 10	**1 + 2 + 3 + 4 = 10. The 4th triangle number is 10.**

Triangle and square numbers are linked. Two triangles can fit together to make a square.

TRY IT

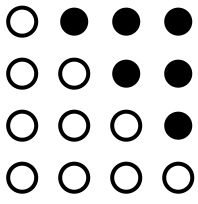
Fit two triangles into a square

You need: 16 stones, beans or bottle tops, a pencil and paper.

1. Make a triangle of 10. Put rows of 1, 2, 3 and 4 stones. Line up the left ends.
2. Make a triangle of 6 in another place. Put rows of 3, 2 and 1 stones.
3. Slide the triangle of 6 into the gaps. Line up the right ends.
4. Count the rows and the columns. Write 10 + 6 = 16.

What you will see: The two triangles fit with no gap. They make a square of 4 rows of 4. So 10 + 6 = 16, a square number.

If you cannot try it, this is what happens.



10 + 6 = 16

It works again with 3 + 6 = 9. And with 10 + 15 = 25. Any two triangle numbers next to each other make a square number.

A WRONG IDEA MANY PEOPLE HAVE

~~**Square numbers go up in equal steps, like 1, 4, 7, 10.**~~

Why it feels right: From 1 to 4 you add 3. It seems you add 3 again.

The test that breaks it: Put 3 rows of 3 stones. Count them. There are 9, not 7. From 4 to 9 you add 5.

What is true: The steps get bigger: 3, 5, 7, 9. Each step is the next odd number.

KEY WORDS

square number
the number of dots in a square of dots.

triangle number
the number of dots in a triangle of dots.

BACK TO YOUR GUESS

The answer is B: That many dots make a square, with as many rows as columns.

A square number is the dots of a square. 4 is 2 rows of 2. 9 is 3 rows of 3. They do not go up by the same step.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. A square has 5 rows of 5 dots. How many dots is that?

   Answer at the back of the book.

2. Which number is a square number?

   A 10

   B 12

   C 16

   Answer at the back of the book.

3. The triangle numbers go 1, 3, 6, 10. What are the next two?

   Answer at the back of the book.

4. The 5th square number is 25. The new dots are 3, 5, 7, 9, then 11. What is the 6th square number?

   Answer at the back of the book.

5. Put the triangle numbers 15 and 21 together. Which square number do you get?

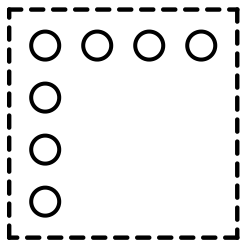
   Answer at the back of the book.

6. **From an earlier chapter**

   Find the step and the missing number: 6, 11, ?, 21.

   Answer at the back of the book.

7. Mei makes a square with 4 rows of 4 dots. How many dots does it have?



4 rows of 4 dots

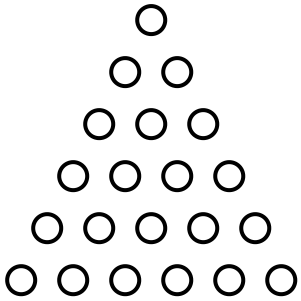
   A 16

   B 8

   C 7

   Answer at the back of the book.

8. The triangle numbers are 1, 3, 6, 10, 15, 21. Which number comes next?



6 rows: 21 dots

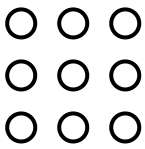
A 27

B 28

C 22

Answer at the back of the book.

9. A square has 3 rows of 3 dots. You add dots to make 4 rows of 4. How many dots do you add?



3 rows of 3 dots

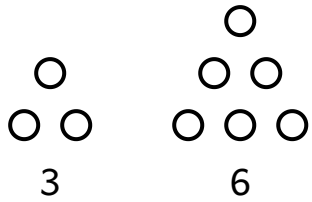
A 16

B 4

C 7

Answer at the back of the book.

10. Two triangles fit to make a square. One has 3 dots and one has 6. How many dots are in the square?



A 9

B 18

C 6

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Square numbers: 1, 4, 9, 16, 25. Triangle numbers: 1, 3, 6, 10, 15, 21. Each new row has one more dot than the last. Triangles of 6 and 10 fit into a square of 16.

Draw: Draw a square of 16 dots. Circle a triangle of 10 inside it. The dots left make a triangle of 6.

M2.8.7 CHAPTER 7

Jumping far ahead

How many sticks do 10 squares need? Must I draw all ten?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Yes. I must draw every square and count.
- ☐ **B** No. A rule gives the answer: 3 × 10 + 1 = 31.
- ☐ **C** Double the sticks for 5 squares. That is 32.

BY THE END YOU CAN

- Find the 10th shape or number without drawing them all
- Write a pattern rule in words and test it

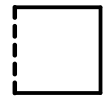
You made squares from sticks in a row. 1 square needs 4 sticks. 2 squares need 7.

Drawing 10 squares is slow. There is a quicker way. Look for a rule. A rule gets you the answer from the **position**. That is the place in the pattern, such as 1st or 10th.

DRAW IT FIRST

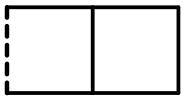
Find the rule for stick squares

3 × 1 + 1 = 4



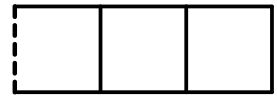
1 One square is 4 sticks. Think of it as 1 start stick, then 3 more.

3 × 2 + 1 = 7



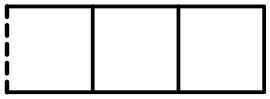
2 A second square adds only 3. It shares a stick with the first.

3 × 3 + 1 = 10



3 A third square adds 3 more. 3 × 3 + 1 = 10 sticks.

10 squares: 3 × 10 + 1 = 31



4 Every square adds 3. So 10 squares need 3 × 10 + 1 = 31 sticks.

The rule in words: take the number of squares. Times it by 3. Then add 1.

Squares	Sticks
1	4
2	7
3	10
4	13
5	16
10	31

Here the position is the number of squares. The last row used the rule.

Test a rule on squares you already know. 3 × 1 + 1 = 4. 3 × 2 + 1 = 7. 3 × 4 + 1 = 13. All three match the table.

The picture shows why. There is 1 start stick, then 3 for each square. So you can trust the rule.

WORKED EXAMPLE

How many sticks do 10 squares need?

1. Each square adds 3 sticks. 10 squares give 3 × 10 = 30 sticks.
2. Add the 1 start stick. 30 + 1 = 31.
3. Check by adding 3 nine times to 4. That also gives 31.

Answer: 31 sticks

Rules work for number patterns too. Take 5, 8, 11, 14. The 1st number is 5. The step is 3.

WORKED EXAMPLE

Find the 10th number of 5, 8, 11, 14.

1. The 1st number is 5. The step is 3.
2. From the 1st to the 10th there are 9 steps, not 10.
3. 9 steps of 3 is 9 × 3 = 27.
4. 5 + 27 = 32. The 10th number is 32.

Answer: 32

Here is a second rule for the same numbers. Times the position by 3, then add 2.

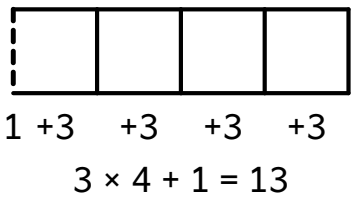
Test it. 3 × 1 + 2 = 5. 3 × 2 + 2 = 8. 3 × 3 + 2 = 11. For the 10th: 3 × 10 + 2 = 32. Both rules agree.

SEE IT THREE WAYS

With things

Make 4 squares in a row with sticks. Count 1 start stick. Then count 3 for each square.

As a picture



3 × 4 + 1 = 13

In symbols

3 × 4 + 1 = 13. So 4 squares need 13 sticks.

TRY IT

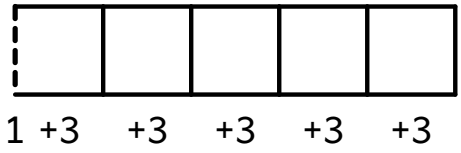
Test the rule with sticks

You need: 16 sticks, straws or strips of paper, a pencil and paper.

1. Make 1 square. Count the sticks. You should count 4.
2. Add 3 sticks to make 2 squares. Count 7.
3. The rule says 5 squares need 3 × 5 + 1 = 16 sticks. Write it down.
4. Now make 5 squares in a row. Count all the sticks.

What you will see: You count 16 sticks. The rule gave the same number without any sticks.

If you cannot try it, this is what happens.



3 × 5 + 1 = 16

A WRONG IDEA MANY PEOPLE HAVE

~~5 squares need 16 sticks, so 10 squares need double: 32.~~

Why it feels right: Twice as many squares seems to need twice as many sticks.

The test that breaks it: Make 1 square. It needs 4 sticks. Make 2 squares. They need 7 sticks, not 8.

What is true: Doubling counts the shared stick twice. Use the rule: 3 × 10 + 1 = 31.

KEY WORDS

position
the place in the pattern, such as 1st, 2nd or 10th.

BACK TO YOUR GUESS

The answer is B: No. A rule gives the answer: 3 × 10 + 1 = 31.

A rule jumps ahead. Each square adds 3 sticks, plus 1 to start. 3 × 10 + 1 = 31. Doubling 16 gives 32, which is too many.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Stick squares need 4 sticks, then 7, then 10. How many sticks do 12 squares need?

   Answer at the back of the book.

2. Find the 10th number of 2, 6, 10, 14.

   A 38

   B 40

   C 42

   Answer at the back of the book.

3. Mei tests the rule "times by 2, then add 1". Does it work for 3 squares of sticks?

   Answer at the back of the book.

4. Ana knows 10 squares need 31 sticks. How many sticks do 20 squares need?

   A 60

   B 61

   C 62

   Answer at the back of the book.

5. A pattern is 3, 8, 13, 18. Find the 8th number.

   Answer at the back of the book.

6. **From an earlier chapter**

   A number machine does "times 3, then add 1". What comes out when 8 goes in?

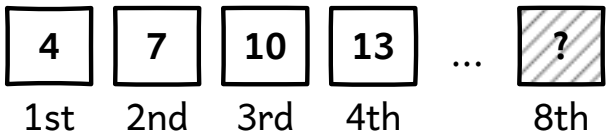
   A 24

   B 25

   C 27

   Answer at the back of the book.

7. Find the 8th number of 4, 7, 10, 13.



A 28

B 25

C 24

Answer at the back of the book.

8. Positions 1 to 4 hold 4, 7, 10 and 13. Which rule gives them?

Position	Number
1	4
2	7
3	10
4	13

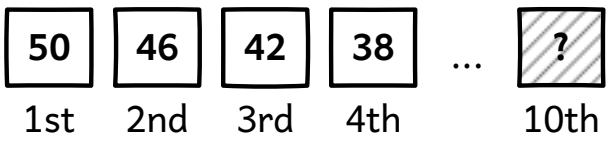
A Times the position by 3.

B Add 3 to the position.

C Times the position by 3, then add 1.

Answer at the back of the book.

9. A pattern goes 50, 46, 42, 38. Find the 10th number.



A 14

B 10

C 86

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

A rule lets you jump ahead. Stick squares: times the number of squares by 3, then add 1. 10 squares need 31 sticks. Test a rule on squares you know, and see why it works.

Draw: Draw 3 squares of sticks in a row. Mark the start stick. Write 3 × 3 + 1 = 10 under them.

Key words, A to Z

Every key word in this book, what it means, and the chapter that teaches it.

even
a number that makes pairs with none left over. M2.8.5

growing pattern
a pattern where each shape is bigger than the shape before. M2.8.2

input
the number you put in. M2.8.3

missing number
a hidden number, which you find. M2.8.4

number machine
a rule that changes a number, one job at a time. M2.8.3

odd
a number that has one left over when you make pairs. M2.8.5

opposite
the job that undoes another. + and − are opposites. M2.8.3

opposite operation
the job that undoes another job. M2.8.4

output
the number that comes out. M2.8.3

position
the place in the pattern, such as 1st, 2nd or 10th. M2.8.7

rule
something that is true every time. M2.8.5

sequence
a list of numbers that follows a rule. M2.8.1

square number
the number of dots in a square of dots. M2.8.6

step
how much the numbers go up or down each time. M2.8.1

table
numbers in rows and columns, such as squares and sticks. M2.8.2

triangle number

the number of dots in *a* tri*a*ngle of dots. M2.8.6

Answers

M2.8.1. Number sequences

1. **26**

   The step is 5, because 11 − 6 = 5. 16 − 11 = 5 and 21 − 16 = 5. So 21 + 5 = 26.

   - **If you chose B:** You counted on by 1. Find the step first: 11 − 6 = 5.
   - **If you chose C:** You doubled 21. Doubling fails on the first pair: 6 doubled is 12, not 11. The step is 5.

2. **50, 40**

   The numbers go down by 10 each time. 60 − 10 = 50 and 50 − 10 = 40.

3. **20**

   The last two numbers give the step. 32 − 26 = 6. Then 14 + 6 = 20, and 20 + 6 = 26.

4. **No. The first jump is 2 and the second is 3.**

   4 − 2 = 2 but 7 − 4 = 3. The steps are not the same. So 2, 4, 7 has no equal step.

   - **If you chose B:** You checked only the first pair. Check every pair: 7 − 4 = 3, not 2.

5. **100, 80, 60, 40, 20**

   Take 20 away each time. 100 − 20 = 80, 80 − 20 = 60, 60 − 20 = 40, 40 − 20 = 20.

6. **23**

   The numbers go down by 6 each time. 47 − 41 = 6 and 41 − 35 = 6. So 29 − 6 = 23.

   - **If you chose A:** You went up by 6. These numbers go down, so take 6 away.
   - **If you chose B:** You took away 12. That is two jumps in one: 47 − 35 = 12. One jump is 47 − 41 = 6.

7. **21**

   The step is 4, because 9 − 5 = 4. 13 − 9 = 4 and 17 − 13 = 4. So 17 + 4 = 21.

   - **If you chose A:** You counted 5, 6, 7, 8, 9. That is 5 numbers but only 4 jumps. The step is 9 − 5 = 4.
   - **If you chose C:** You doubled 17. Doubling fails on the first pair: 5 doubled is 10, not 9.

8. **7**

   Take one number from the next: 21 − 14 = 7. Check: 28 − 21 = 7 and 35 − 28 = 7.

   - **If you chose A:** You took 14 from 35. That is three steps together. A step is one jump, from one mark to the next.
   - **If you chose B:** You used the first number. The step is the jump from one number to the next.

9. **11**

   The last two marks give the step: 23 − 17 = 6. The gap is one step after 5: 5 + 6 = 11. Check: 11 + 6 = 17.

   - **If you chose B:** A step of 5 does not fit: 10 + 5 = 15, not 17. Use the last two marks: 23 − 17 = 6.
   - **If you chose C:** You took 5 from 17. That is two hops, the gap and the one after it. One hop is 23 − 17 = 6.

M2.8.2. Growing shape patterns

1. **19 sticks**

   4, 7, 10, 13, 16, 19. Or take 3 times 6, then add 1. 3 × 6 + 1 = 19.

2. **No. They need 10 sticks.**

   The first square needs 4. The next two add 3 each. 4 + 3 + 3 = 10.

   - **If you chose B:** You gave every square 4 sticks. Joined squares share sticks, so each new one adds only 3.
   - **If you chose C:** You used 3 sticks for every square. The first square needs 4 sticks.

3. **10, 13, 16**

   Add 3 each time. 7 + 3 = 10, 10 + 3 = 13, 13 + 3 = 16.

4. **4 squares in a row, with 13 sticks**

   Three squares use 10 sticks. The fourth square adds 3. 10 + 3 = 13.

5. **13**

   The step is 4, because 21 − 17 = 4. 9 + 4 = 13, and 13 + 4 = 17.

6. **Shape A**

   Each shape adds 1 square to the end of the row. After 3 comes 4. Shape A has 4 squares in a row.

   - **If you chose B:** The pattern adds one square to the end of the row. It does not start a new row.
   - **If you chose C:** Each new square joins the end of the row, not the top.

7. **4 squares: 12**

   The sticks go up by 3 each row. 10 + 3 = 13, not 12. The 4 squares row should say 13.

   - **If you chose A:** This row is right: 13 + 3 = 16. The row before it is the one that breaks the step.
   - **If you chose B:** 3 × 4 = 12 counts the shared sticks twice. Joined squares share, so 7 + 3 = 10 is right.

8. **31**

   Take 3 times the squares, then add 1. 3 × 10 + 1 = 31. Check: 4 + 3 × 9 = 31.

   - **If you chose A:** You did 3 times 10 and forgot the 1 start stick. Add 1.
   - **If you chose B:** You gave every square 4 sticks. Joined squares share sticks.
   - **If you chose C:** You added 3 ten times. The first square is 4 sticks. Only 9 squares add 3.

9. **6**

   Count up the table. 4, 7, 10, 13, 16, 19. That is 6 squares. Check: 3 × 6 + 1 = 19.

   - **If you chose B:** You counted the steps of 3. 4 + 5 × 3 = 19. The first square is not a step, so count it too.
   - **If you chose C:** You shared 19 sticks by 4. Joined squares share sticks, so 4 squares need only 13.

M2.8.3. Number machines

1. **28**

   First job: 6 × 5 = 30. Second job: 30 − 2 = 28.

2. **20**

   First job: 3 + 2 = 5. Second job: 5 × 4 = 20.

   - **If you chose B:** You did × 4 first. 3 × 4 + 2 = 14. This machine adds 2 first.
   - **If you chose C:** You added 2 and then added 4. The second job is × 4, not + 4.

3. **× 4, then + 1**

   The inputs go up by 1 and the outputs by 4. So start with × 4. 2 × 4 = 8, and the output is 9, so add 1. Test: 3 × 4 + 1 = 13 and 4 × 4 + 1 = 17.

4. **7**

   Undo + 5 first. 19 − 5 = 14. Then undo × 2. 14 ÷ 2 = 7. Check: 7 × 2 + 5 = 19.

5. **22 sticks**

   4 sticks for the first square. Then 6 more squares add 3 each. 6 × 3 = 18, and 4 + 18 = 22. Or 3 × 7 + 1 = 22.

6. **8**

   Undo the last job first. 20 − 4 = 16. Then undo × 2. 16 ÷ 2 = 8. Check: 8 × 2 + 4 = 20.

   - **If you chose A:** You undid × 2 first: 20 ÷ 2 = 10, then 10 − 4 = 6. Undo the last job first, which is + 4.
   - **If you chose C:** You did + 4 and × 2 again. To go back, use the opposite of each job: − 4 and ÷ 2.

7. **× 3, then + 2**

   The outputs go up by 3, so start with × 3. 1 × 3 = 3, and the output is 5, so add 2. Test: 2 × 3 + 2 = 8 and 3 × 3 + 2 = 11.

   - **If you chose A:** The outputs go up by 3, but 1 + 3 = 4, not 5. A rule must fit every pair.
   - **If you chose B:** You used the first output, 5. 1 × 5 = 5 fits, but 2 × 5 = 10, not 8.

8. **17**

   First job: 5 × 4 = 20. Second job: 20 − 3 = 17.

   - **If you chose A:** You added 3. The second job is − 3, so take 3 away.
   - **If you chose C:** You took 3 away first, then did × 4. The jobs go in order: × 4 comes first.

9. **18**

   Undo the last job first. 15 − 6 = 9. Then undo ÷ 2. 9 × 2 = 18. Check: 18 ÷ 2 + 6 = 15.

   - **If you chose A:** You undid ÷ 2 first: 15 × 2 = 30, then 30 − 6 = 24. Undo the last job first, which is + 6.
   - **If you chose B:** You undid + 6 but not ÷ 2. The opposite of ÷ 2 is × 2, so 9 × 2 = 18.

M2.8.4. Missing numbers

1. **42**

   Take the known part from the whole. 71 − 29 = 42. Check: 29 + 42 = 71.

2. **7**

   The opposite of × is ÷. 56 ÷ 8 = 7. Check: 7 × 8 = 56.

3. **No. 25 + 85 = 110. The missing number is 35.**

   The whole is 60. 60 − 25 = 35. Check: 25 + 35 = 60.

   - **If you chose B:** You added the two numbers you can see. Put 85 back to check: 25 + 85 is far more than 60.
   - **If you chose C:** You took the smaller digit from the bigger in each place. Put 45 back: 25 + 45 = 70, not 60.

4. **24 beans**

   48 + ? = 72. The opposite of + is −. 72 − 48 = 24. Check: 48 + 24 = 72.

5. **7**

   Undo + 3 first. 31 − 3 = 28. Then undo × 4. 28 ÷ 4 = 7. Check: 7 × 4 + 3 = 31.

6. **27**

   The whole is 80. Take the part you know away. 80 − 53 = 27. Check: 53 + 27 = 80.

   - **If you chose A:** You took the smaller digit from the bigger in each place. Put 33 back: 53 + 33 = 86, not 80.
   - **If you chose B:** You added the two numbers you can see. The part must be less than the whole, 80.

7. **7**

   The opposite of × is ÷. 63 ÷ 9 = 7. Check: 7 × 9 = 63.

   - **If you chose A:** You took 9 away from 63. The opposite of × is ÷, not −.
   - **If you chose C:** 8 × 9 = 72. That is one 9 too many. 7 × 9 = 63.

8. **36 + 14 = 50**

   The opposite of − is +. Put the answer back with the part you know. 36 + 14 = 50, the whole.

   - **If you chose A:** You added the whole and the answer. Add the answer to the part you took away.
   - **If you chose C:** You took the answer from the part. Add them instead, because + undoes −.

9. **9**

   Share 54 into 6 equal bags. 54 ÷ 6 = 9. Check: 6 × 9 = 54.

   - **If you chose B:** You took 6 away from 54. The bags are equal, so share 54 into 6 groups with ÷.
   - **If you chose C:** 6 × 8 = 48. That is 6 short of 54. 6 × 9 = 54.

M2.8.5. Odd and even rules

1. **7 + 8**

   7 is odd and 8 is even. Only 7 has a left over. So 7 + 8 = 15 is odd.

   - **If you chose A:** 6 and 8 are both even. They make pairs with none left over. 6 + 8 = 14, which is even.
   - **If you chose B:** You may think two odd numbers make an odd number. Each has one left over. The two left overs make a pair. 7 + 9 = 16.

2. **even**

   13 and 27 are both odd. Odd + odd is even. Check: 13 + 27 = 40, which ends in 0.

3. **odd**

   9 has one left over. 6 has none. So one dot has no partner. 9 + 6 = 15, which is odd.

4. **odd**

   5 + 7 = 12 is even. 12 + 9 is even + odd, so it is odd. 12 + 9 = 21.

5. **Test more pairs, and ask why it works.**

   Examples help you trust a rule. One that fails breaks it. A picture of pairs shows why it works.

   - **If you chose A:** Three examples are a start, but they do not prove a rule. Test more pairs, and ask why it works.
   - **If you chose C:** Many examples build trust. A picture of pairs can show why a rule works.

6. **32**

   Take 29 from 61: 61 − 29 = 32. Check: 29 + 32 = 61.

7. **The two left overs make a pair. The total is even.**

   Each odd number has one left over. The two left overs make a pair. So 5 + 3 has none left over. 5 + 3 = 8, which is even.

   - **If you chose A:** Each left over now has a partner, the other left over. They make a pair.
   - **If you chose C:** The total needs both numbers. 3 has a left over too. The two left overs make a pair: 5 + 3 = 8.

8. **Odd. Two left overs make a pair, but the third stays alone.**

   Each of 3, 5 and 7 has one left over. Two left overs make a pair. The third stays alone. 3 + 5 + 7 = 15, which is odd.

   - **If you chose B:** That rule is for two odd numbers. Here there are three. 3 + 5 = 8, and 8 + 7 = 15, which is odd.
   - **If you chose C:** You can tell. 3 + 5 is even, and even + odd is odd.

9. **No. 3 + 4 = 7 breaks it.**

   7 is odd. One example that fails is enough to break a rule. Tests that work only build trust.

   - **If you chose A:** 3 + 5 = 8 is even. An example that fits the rule cannot break it.
   - **If you chose B:** Tests that work do not prove a rule. Try 3 + 4. It makes 7, which is odd.

M2.8.6. Square and triangle numbers

1. **25**

   5 rows of 5 dots is 5 × 5 = 25. Check with the table: the 5th square number is 25.

2. **16**

   16 is 4 × 4. It makes a square of 4 rows of 4.

   - **If you chose A:** 10 is a triangle number: 1 + 2 + 3 + 4. It makes a triangle of dots, not a square.
   - **If you chose B:** 12 is 3 × 4. The rows and columns differ. The dots make a rectangle.

3. **15 and 21**

   The 5th row has 5 dots. 10 + 5 = 15. The 6th row has 6 dots. 15 + 6 = 21.

4. **36**

   25 + 11 = 36. Check: 6 rows of 6 is 6 × 6 = 36.

5. **36**

   Two triangle numbers next to each other make a square number. 15 + 21 = 36, which is 6 × 6.

6. **The step is 5. The missing number is 16.**

   11 − 6 = 5. So the missing number is 11 + 5 = 16. Check: 16 + 5 = 21. This pattern goes up by the same step each time. Square numbers do not.

7. **16**

   4 rows of 4 dots is 4 × 4 = 16. The 4th square number is 16.

   - **If you chose B:** You added 4 + 4. A square has 4 rows, so you need 4 × 4.
   - **If you chose C:** You counted only the dots drawn. The dashed square has 4 rows, and every row has 4 dots.

8. **28**

   The next row is the 7th row. It has 7 dots. 21 + 7 = 28.

   - **If you chose A:** You added 6 again, like the last jump. The 7th row has 7 dots.
   - **If you chose C:** You added 1 dot. The 7th row has 7 dots, so add 7.

9. **7**

   The new square has 4 × 4 = 16 dots. You have 3 × 3 = 9. So you add 16 − 9 = 7 dots.

   - **If you chose A:** That is how many dots the new square has. You already have 9 of them.
   - **If you chose B:** You added one row of 4. You must add a column of 3 as well.

10. **9**

    Add the dots of both triangles. 3 + 6 = 9. It is 3 rows of 3. 9 is a square number.

    - **If you chose B:** You multiplied 3 × 6. Two triangles that fit together keep all their dots, so add them.
    - **If you chose C:** You counted only the bigger triangle. The square holds the dots of both triangles.

M2.8.7. Jumping far ahead

1. **37 sticks**

   Each square adds 3, plus 1 to start. 3 × 12 + 1 = 37.

2. **38**

   The 1st number is 2. There are 9 steps of 4. 9 × 4 = 36. 2 + 36 = 38.

   - **If you chose B:** You may have done 10 × 4. But the 1st number is 2, not 4. Test your rule on the 1st number.
   - **If you chose C:** You added 4 ten times. From the 1st to the 10th there are 9 steps, not 10.

3. **No. It gives 7, but 3 squares need 10 sticks.**

   2 × 3 + 1 = 7. The table says 3 squares need 10. The rule that works is 3 × 3 + 1 = 10.

4. **61**

   Each square adds 3. 3 × 20 = 60. Add the start stick: 61.

   - **If you chose A:** You did 3 × 20 and forgot the start stick. Add 1.
   - **If you chose C:** You doubled 31. That counts the shared stick twice. Use 3 × 20 + 1.

5. **38**

   The 1st number is 3. There are 7 steps of 5. 7 × 5 = 35. 3 + 35 = 38. Rule check: 5 × the position, take away 2. 5 × 8 − 2 = 38.

6. **25**

   8 × 3 = 24. Then 24 + 1 = 25.

   - **If you chose A:** You did times 3 but forgot to add 1. Go to M2.8.3.
   - **If you chose C:** You added 1 first, then times 3. The order matters. Times 3 comes first. Go to M2.8.3.

7. **25**

   The 1st number is 4. There are 7 steps of 3. 7 × 3 = 21. 4 + 21 = 25.

   - **If you chose A:** You added 3 eight times. From the 1st to the 8th there are 7 steps, not 8.
   - **If you chose C:** You did 8 × 3, but the 1st number is 4, not 3. Test a rule on the 1st number.

8. **Times the position by 3, then add 1.**

   The numbers go up by 3, so start with × 3. 1 × 3 = 3, and the 1st number is 4, so add 1. Test: 2 × 3 + 1 = 7 and 4 × 3 + 1 = 13.

   - **If you chose A:** 1 × 3 = 3, but the 1st number is 4. Test a rule on the 1st number.
   - **If you chose B:** 1 + 3 = 4 fits, but 2 + 3 = 5, not 7. A rule must fit every position.

9. **14**

   The 1st number is 50. There are 9 steps of 4 down. 9 × 4 = 36. 50 − 36 = 14.

   - **If you chose B:** You took 4 away ten times. From the 1st to the 10th there are 9 steps, not 10.
   - **If you chose C:** You added 4 nine times. This pattern goes down, so take 4 away each time.

About this book

For learners who can add, take away and use the times tables to 10. In many schools this fits children of about 8 to 10, and older learners can start here too. Activities use sticks, beans and paper. This book prepares the ground for letters standing for

numbers at Level 3. A learner can work alone: every exercise has its answer and the reason for it.

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