●●●●○ Level 4 · Mathematics M4.5

Area, volume and compound units

Find the volume and surface area of solids like cylinders, cones and spheres, and work with speed and density.

Find the volume of prisms and cylinders, then of pyramids, cones, spheres and joined or cut solids. Change between units of area and volume. Use speed, density, pressure and rates of flow, and read what a rate graph says.

YOU ARE READY FOR THIS BOOK IF

You can find the area of a circle with a radius of 5 cm.

This book	**M4.5 Area, volume and compound units**
One step down	M3.6 Perimeter, area and volume

Contents

M4.5.1 Prisms and cylinders

M4.5.2 Pyramids and cones

M4.5.3 Spheres

M4.5.4 Joined and cut solids

M4.5.5 Changing units of area and volume

M4.5.6 Speed, distance and time

M4.5.7 Density and pressure

M4.5.8 Rates of flow and rate graphs

Key words, A to Z

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M4.5.1 CHAPTER 1

Prisms and cylinders

How much does a tin hold, and how much metal is in it?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Multiply the three measurements you can see.
- ☐ **B** Multiply the area of the end face by the length.
- ☐ **C** Add the area of the end face to the length.

IF THIS FEELS HARD, READ THIS FIRST

- Level 3, Perimeter, area and volume, M3.6.7. If volume as length × width × height looks new, read this first.
- Level 3, Perimeter, area and volume, M3.6.6. If the area and circumference of a circle look new, read this first.
- Level 3, Perimeter, area and volume, M3.6.8. If finding surface area from a net looks new, read this first.

BY THE END YOU CAN

- Find the volume of a prism as end area × length
- Find the volume and surface area of a cylinder

A loaf tin, a roof beam and a tin of beans have one thing in common. Cut any of them across and you get the same flat shape, wherever you cut.

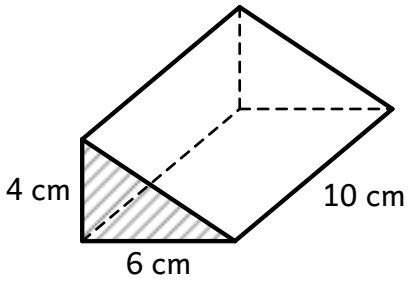
That flat shape is the **cross section**. A **prism** is a solid whose cross section stays the same all the way along. A cylinder works the same way, with a circle as its cross section.

You already know one prism. A box with a rectangle at the end has volume length × width × height. The length × width part is the area of the end face. So for a box,

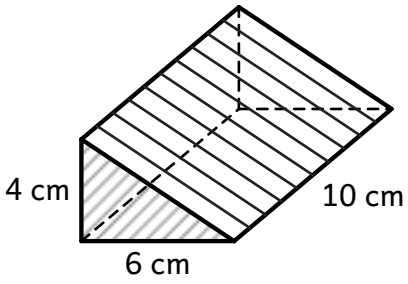
volume = end area × length.

DRAW IT FIRST

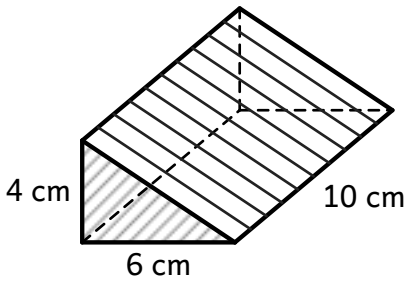
Slice a prism



1 Here is a prism. Its end face is a triangle, and it is 10 cm long.



2 Cut it into 10 slices, each 1 cm thick. Every slice has the same end face.



each 1 cm slice: 12 cm^3

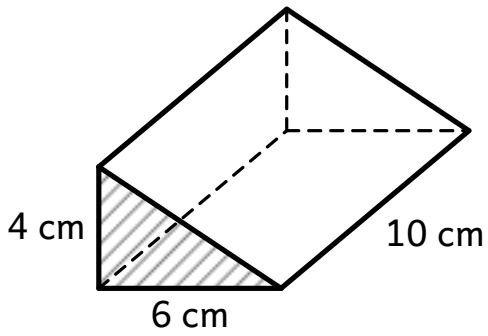
3 Each slice holds 12 cm^3, the end area of 12 cm^2 times 1 cm. Ten of them make 120 cm^3.

In these pictures, the edges that go back into the page are drawn shorter. This makes the solid look solid. Read lengths from the labels, not from the drawing.

The slices show why the rule works. Make the slices thinner and thinner and the rule does not change. The volume is the end area, counted once for each unit of length.

For a prism whose cross section has area A and whose length is l, the volume is V = A × l. The rule needs a cross section that stays the same. If the shape changes along the solid, it is not a prism.

Watch the units. If A is in cm^2 and l is in cm, then V is in cm^3. A volume is an area in cm^2, stretched along a length in cm.



A triangular prism. The shaded end face is the cross section.

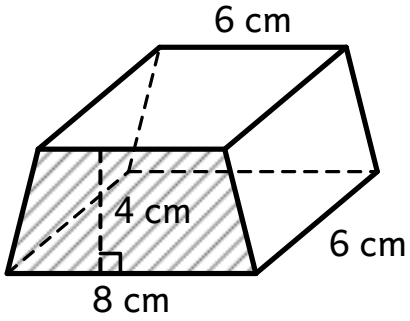
WORKED EXAMPLE

Find the volume of the prism in the picture. The end face is a right angled triangle with base 6 cm and height 4 cm. The prism is 10 cm long.

1. The end face is a triangle. Its area is ½ × 6 × 4 = 12 cm^2.
2. The prism is 10 cm long, and the end face stays the same all the way along.
3. Volume = end area × length = 12 × 10 = 120 cm^3.

Answer: 120 cm^3

The end face can be any shape you can find the area of. Here it is a trapezium. Find the area of the end face first, then multiply by the length.



A trapezium prism. The dashed line is the height of the trapezium.

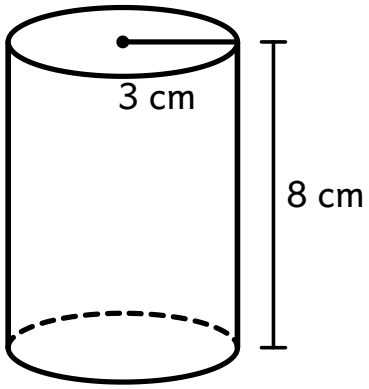
WORKED EXAMPLE

Find the volume of the trapezium prism in the picture. Its parallel sides are 8 cm and 6 cm, and they are 4 cm apart. The prism is 6 cm long.

1. The end face is a trapezium. Its area is $\frac{1}{2} \times (8 + 6) \times 4 = 28$ cm^2.
2. The prism is 6 cm long, and the end face stays the same all the way along.
3. Volume = end area × length = $28 \times 6 = 168$ cm^3.

Answer: 168 cm^3

Now the cylinder. Its cross section is a circle of radius r, with area πr^2. So the volume of a cylinder of height h is $V = \pi r^2 \times h$.



A cylinder with radius 3 cm and height 8 cm.

WORKED EXAMPLE

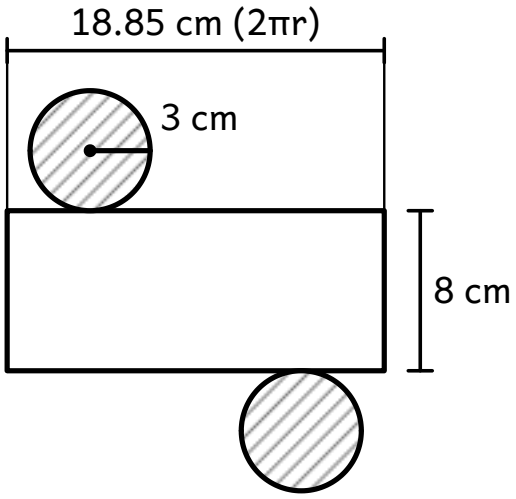
Kofi has a tin shaped like a cylinder, with radius 3 cm and height 8 cm. Find its volume, to 1 decimal place.

1. The end face is a circle of radius 3 cm. Its area is $\pi \times 3^2 = 9\pi$ cm^2.
2. Volume = end area × height = $9\pi \times 8 = 72\pi$ cm^3.
3. Put in the π key of a calculator. To 2 decimal places, 72π is 226.19 cm^3.
4. To 1 decimal place, the volume is 226.2 cm^3.

Answer: 226.2 cm^3

Surface area is the area of all the faces added up. To see a cylinder's faces, cut it open and lay it flat. This flat picture is its **net**.

The net has two circles and one rectangle. The rectangle wraps once round the circle, so one side is the height h and the other side is the circumference $2\pi r$. That rectangle is the **curved surface**.



The net of the cylinder. The rectangle is as long as the circumference of the circle.

The curved surface has area $2\pi r \times h = 2\pi rh$. Each circle has area πr^2, and there are two. The total surface area of a closed cylinder is $2\pi rh + 2\pi r^2$.

WORKED EXAMPLE

Find the total surface area of the cylinder with radius 3 cm and height 8 cm. Give the answer to 1 decimal place.

1. Cut the cylinder open. The curved surface is a rectangle as long as the circumference of a circle.
2. The circumference is $2 \times \pi \times 3 = 6\pi$ cm. The curved surface is $6\pi \times 8 = 48\pi$ cm^2.
3. Each end is a circle of area $\pi \times 3^2 = 9\pi$ cm^2. The two ends make 18π cm^2.
4. The total surface area is $48\pi + 18\pi = 66\pi$ cm^2.
5. Put in the π key of a calculator. To 1 decimal place, 66π is 207.3 cm^2.

Answer: 207.3 cm^2

TRY IT

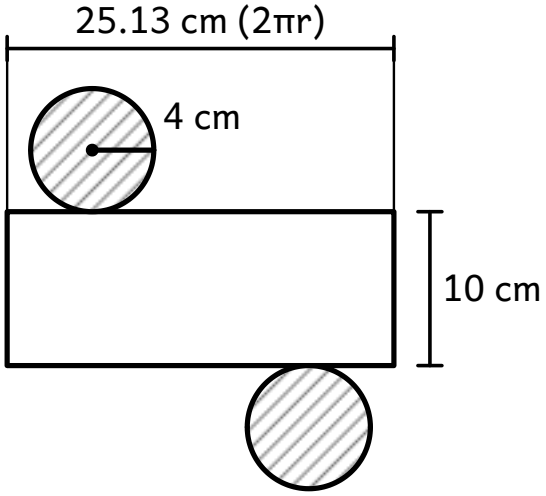
Wrap a tin

You need: a round tin or a cup, a strip of paper, a pencil and a ruler.

1. Wrap the paper once round the tin. Mark where the paper meets itself.
2. Unwrap it and measure from the end to the mark. This is the circumference.
3. Measure straight across the top of the tin. This is the diameter.
4. Divide the circumference by the diameter.

What you will see: The answer is a little over 3 for every round tin, big or small. That number is π. It is why the paper rectangle is 2πr long.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~The volume of a cylinder is π × diameter^2 × height.~~

Why it feels right: The diameter is the width you measure across a tin, so it feels like the number to use.

The test that breaks it: Take a cylinder with radius 3 cm and height 8 cm. The true volume is π × 9 × 8 = 72π, which is 226.2 cm^3 to 1 decimal place. With the diameter, 6 cm, you get π × 36 × 8 = 288π, which is 904.8 cm^3. That is four times too big.

What is true: The area of a circle is πr^2, with r the radius. Halve the diameter first.

A WRONG IDEA MANY PEOPLE HAVE

~~The volume of a triangular prism is base × height × length.~~

Why it feels right: For a box this is exactly right, and a triangle looks like a box with a corner cut off.

The test that breaks it: Take the prism with an end face of base 6 cm and height 4 cm, 10 cm long. 6 × 4 × 10 = 240 cm^3. But the end face is a triangle, with half the area of the 6 by 4 rectangle. Half of 240 is 120 cm^3, the answer from the slices.

What is true: Find the area of the end face first. A triangle is ½ × base × height.

KEY WORDS

cross section
the flat shape you see when you cut a solid straight across.

prism
a solid whose cross section stays the same all the way along.

net
the faces of a solid laid out flat.

curved surface

the side of a cylinder, without its two ends.

BACK TO YOUR GUESS

The answer is B: Multiply the area of the end face by the length.

A prism is built from the same flat slice, again and again, all the way along. The volume is the area of one slice times the length. Multiplying three measurements only works for a box, and adding an area to a length mixes two kinds of measure.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Which of these solids is a prism?

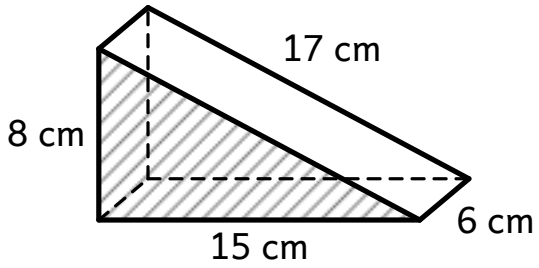
   A A cone

   B A tent shaped like a triangle, with the same triangle at both ends

   C A ball

   Answer at the back of the book.

2. The end face of this prism is a right angled triangle with base 15 cm and height 8 cm. The prism is 6 cm long. What is its volume?



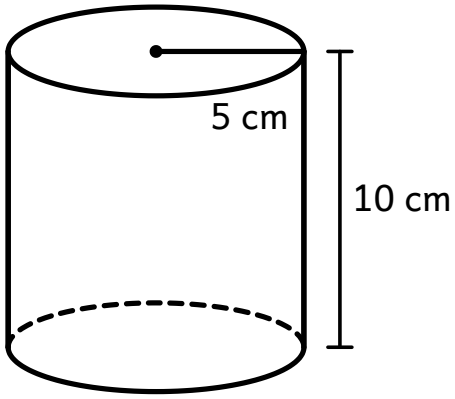
   A 720 cm^3

   B 240 cm^3

   C 360 cm^3

   Answer at the back of the book.

3. A cylinder has radius 5 cm and height 10 cm. What is its volume, to 1 decimal place?



A 785.4 cm^3

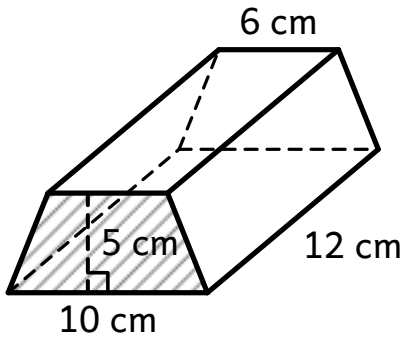
B 3 141.6 cm^3

C 157.1 cm^3

D 314.2 cm^3

Answer at the back of the book.

4. The end face of this prism is a trapezium. Its parallel sides are 10 cm and 6 cm, and they are 5 cm apart. The prism is 12 cm long. What is its volume?



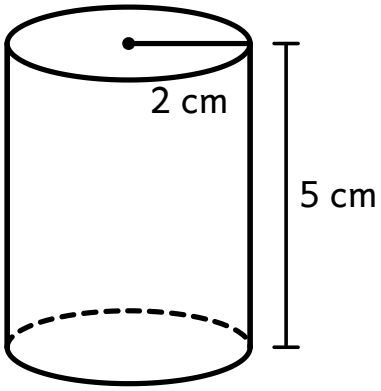
A 960 cm^3

B 480 cm^3

C 600 cm^3

Answer at the back of the book.

5. A closed cylinder has radius 2 cm and height 5 cm. What is its total surface area, to 1 decimal place?



A 62.8 cm^2

B 25.1 cm^2

C 75.4 cm^2

D 88.0 cm^2

Answer at the back of the book.

6. One cylinder has radius 3 cm. A second cylinder has the same height and radius 6 cm. How many times as big is the volume of the second cylinder?

A Twice as big

B 8 times as big

C 4 times as big

Answer at the back of the book.

7. The end face of a prism is a right angled triangle with base 8 cm and height 5 cm. The prism is 9 cm long. What is its volume?

Answer at the back of the book.

8. A cylinder has radius 4 cm and height 7 cm. What is its volume, to 1 decimal place?

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Prism: volume = end area × length. Cylinder: $V = \pi r^2 h$. Curved surface $2\pi rh$. Two ends $2\pi r^2$. Use the radius, not the diameter.

Draw: A cylinder, and beside it its net: a rectangle with two circles, the rectangle as long as $2\pi r$.

M4.5.2 CHAPTER 2

Pyramids and cones

Does a cone hold as much as the cylinder it fits inside?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Half as much as the cylinder.
- ☐ **B** The same amount as the cylinder.
- ☐ **C** One third as much as the cylinder.

IF THIS FEELS HARD, READ THIS FIRST

- Level 4, Area, volume and compound units, M4.5.1. If volume as area of the end face × length looks new, read this first.
- Level 4, Pythagoras and trigonometry, M4.3.2. If finding a slant length with Pythagoras looks new, read this first.
- Level 4, Similarity, transformations and circles, M4.4.7. If the arc of a circle and its sector look new, read this first.
- Level 3, Perimeter, area and volume, M3.6.6. If the area of a circle looks new, read this first.

BY THE END YOU CAN

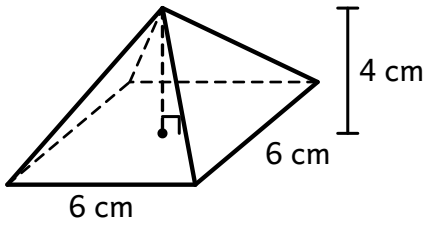
- Find the volume of a pyramid as 1/3 × base area × height
- Find the volume and curved surface area of a cone

A pyramid and a cone both rise from a flat base to a single point, the ***apex***. A tent, a funnel and an ice cream cone all have this shape.

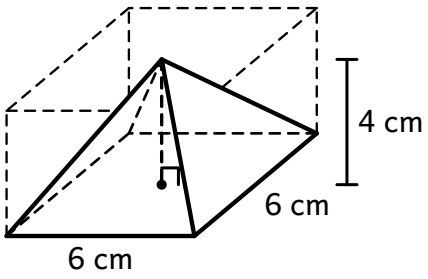
Take a pyramid and a prism with the same base and the same height. The pyramid fits inside the prism. How much of the prism does it fill?

DRAW IT FIRST

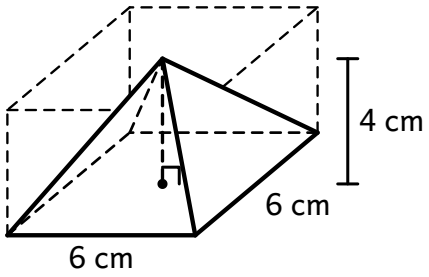
A pyramid inside a prism



1 Draw a pyramid with a square base. Its height is the straight line from the apex down to the base.



2 Draw the box with the same base and the same height round the pyramid.

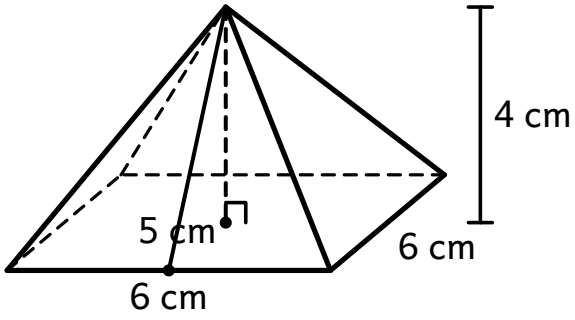


3 pyramids of sand fill the box

3 Three pyramids like this one, poured as sand, fill the box exactly once.

This is a fact you can test with sand, and it can be proved in more advanced mathematics. The pyramid fills one third of the box. The box has volume base area × height. So for a pyramid, V = ⅓ × base area × height.

The height must be the **perpendicular height**. It is measured at a right angle to the base, straight down from the apex. The **slant height** runs down a sloping face. It is longer, and it is not used in this volume.



The perpendicular height is 4 cm. The slant height of the front face is 5 cm.

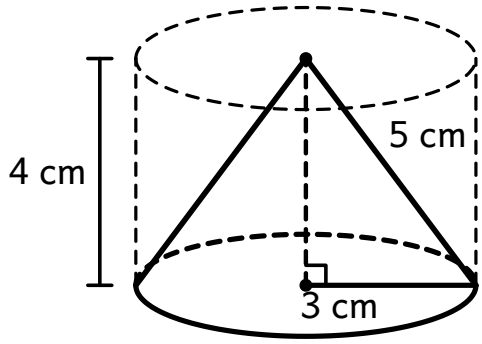
WORKED EXAMPLE

Mei builds a model pyramid with a square base of side 6 cm. Its perpendicular height is 4 cm. Find its volume.

1. The base is a square. Its area is $6 \times 6 = 36$ cm^2.
2. Volume = $\frac{1}{3} \times$ base area $\times$ perpendicular height = $\frac{1}{3} \times 36 \times 4$.
3. $\frac{1}{3} \times 36 \times 4 = 12 \times 4 = 48$ cm^3.

Answer: 48 cm^3

A cone is a pyramid with a circle for its base. The base area is πr^2, so the volume of a cone is $V = \frac{1}{3}\pi r^2 h$, with h the perpendicular height.



The cone fills one third of the cylinder round it.

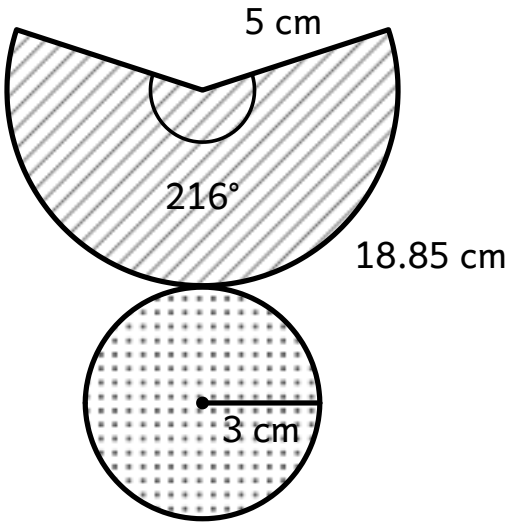
WORKED EXAMPLE

A cone has base radius 3 cm and perpendicular height 4 cm. Find its volume, to 1 decimal place.

1. The base is a circle of radius 3 cm. Its area is $\pi \times 3^2 = 9\pi$ cm^2.
2. Volume = ⅓ × base area × height = ⅓ × 9π × 4.
3. ⅓ × 9π × 4 = 3π × 4 = 12π cm^3.
4. Put in the π key of a calculator. 12π is 37.7 cm^3, to 1 decimal place.

Answer: 37.7 cm^3

Now the curved surface of a cone. Cut it open and lay it flat. You get a sector of a circle, with a circle for the base beside it.



The sector has radius l, the slant height. Its curved edge is the circumference of the base.

The straight sides of the sector are the slant height l. The curved edge is the circumference of the base circle, 2πr. A whole circle of radius l has circumference 2πl and area πl^2.

Our sector is the fraction 2πr ÷ 2πl = r ÷ l of that whole circle. So its area is (r ÷ l) × πl^2 = πrl. The curved surface area of a cone is πrl.

To find l, use Pythagoras on the right angled triangle inside the cone. Its sides are r, the perpendicular height h, and l: l^2 = r^2 + h^2. So l = √(r^2 + h^2).

WORKED EXAMPLE

A cone has base radius 3 cm and perpendicular height 4 cm. Find the area of its curved surface, to 1 decimal place.

1. The slant height l joins the apex to the rim. It is the longest side of a right angled triangle with sides 3 and 4.
2. $l^2 = 3^2 + 4^2 = 9 + 16 = 25$.
3. $l = \sqrt{25} = 5$ cm.
4. Curved surface area $= \pi \times r \times l = \pi \times 3 \times 5 = 15\pi$ cm^2.
5. Put in the π key of a calculator. 15π is 47.1 cm^2, to 1 decimal place.

Answer: 47.1 cm^2

TRY IT

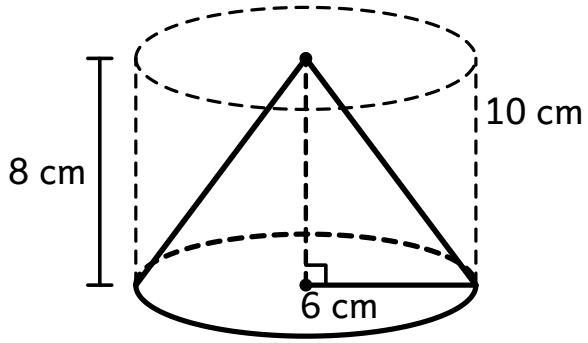
Three cones fill *a* cylinder

You need: *a* sheet of stiff paper, *a* piece of tape, dry rice, sand or salt.

1. Roll the paper into *a* cone and tape it. Cut the wide end flat, so the cone can stand on it.
2. Roll *a* second sheet into *a* cylinder with the same wide end and the same height.
3. Fill the cone with rice and tip it into the cylinder.
4. Count how many cones of rice fill the cylinder.

What you will see: You need three cones. The cone holds one third as much as the cylinder, which is why there is *a* ⅓ in the volume of *a* cone.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~In the volume of *a* cone, h is the slant height.~~

Why it feels right: The slant height is the length you can see along the side, and you can measure it with *a* ruler.

The test that breaks it: Take the cone with radius 3 cm, perpendicular height 4 cm and slant height 5 cm. The true volume is ⅓ × π × 9 × 4 = 12π, which is 37.7 cm^3 to 1 decimal place. With 5 you get ⅓ × π × 9 × 5 = 15π, which is 47.1 cm^3, and that is too big.

What is true: Use the perpendicular height, at *a* right angle to the base. The slant height belongs in the curved surface, πrl.

A WRONG IDEA MANY PEOPLE HAVE

~~The volume of a pyramid is base area × height, like a prism.~~

Why it feels right: A pyramid has a base and a height, and for prisms we multiply them.

The test that breaks it: Take the pyramid with a 6 cm square base and a height of 4 cm. Base area × height is 36 × 4 = 144 cm^3. That is the whole box. Three pyramids fill the box, so one pyramid is 144 ÷ 3 = 48 cm^3.

What is true: A pyramid fills only one third of the prism with the same base and height. Multiply by ⅓.

KEY WORDS

apex
the point at the top of a pyramid or a cone.

perpendicular height
the height from the apex straight down to the base, at a right angle to it.

slant height
the length from the apex down the sloping side to the base edge.

BACK TO YOUR GUESS

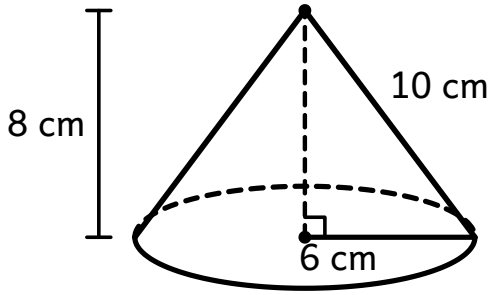
The answer is C: One third as much as the cylinder.

A cone with the same base and height as a cylinder holds exactly one third as much. Fill the cone with rice, tip it into the cylinder, and you need three cones to fill it.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. A cone has radius 6 cm, perpendicular height 8 cm and slant height 10 cm. What is its volume, to 1 decimal place?



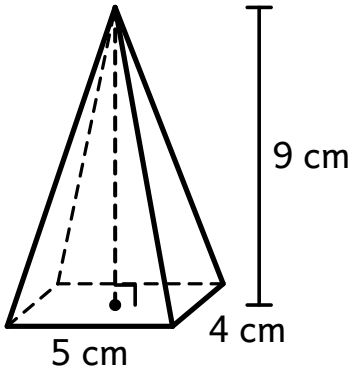
A 377.0 cm^3

B 904.8 cm^3

C 301.6 cm^3

Answer at the back of the book.

2. A pyramid has a rectangular base 5 cm by 4 cm. Its perpendicular height is 9 cm. What is its volume?



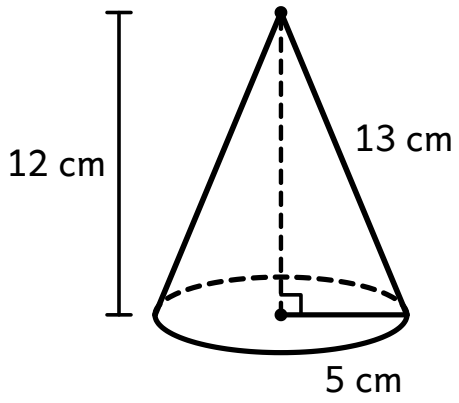
A 180 cm^3

B 60 cm^3

C 90 cm^3

Answer at the back of the book.

3. A cone has radius 5 cm, perpendicular height 12 cm and slant height 13 cm. What is the area of its curved surface, to 1 decimal place?



A 204.2 cm^2

B 188.5 cm^2

C 408.4 cm^2

Answer at the back of the book.

4. A cone and a cylinder have the same base and the same height. The cone holds 30π cm^3 of rice and the cylinder holds 90π cm^3. How many cones full of rice fill the cylinder?

A 2 cones

B 4 cones

C 3 cones

Answer at the back of the book.

5. A cone has radius 6 cm and perpendicular height 8 cm. What is its slant height?

A 14 cm

B 10 cm

C 100 cm

Answer at the back of the book.

6. **From an earlier chapter**

   A cylinder has radius 4 cm and height 5 cm. What is its volume, to 1 decimal place?

   Answer at the back of the book.

7. A pyramid has a square base of side 6 cm and a perpendicular height of 9 cm. What is its volume?

   Answer at the back of the book.

8. A cone has radius 4 cm and perpendicular height 9 cm. What is its volume, to 1 decimal place?

   Answer at the back of the book.

9. A cone has radius 9 cm and slant height 15 cm. What is the area of its curved surface, to 1 decimal place?

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Pyramid and cone: V = ⅓ × base area × perpendicular height. Cone V = ⅓πr²h. Curved surface πrl, with $l^2 = r^2 + h^2$. Not the slant height in V.

Draw: A cone with r, h and l marked, the height as a dashed upright line with a right angle at the base.

M4.5.3 CHAPTER 3

Spheres

> **How much air is in a football, and how much leather covers it?**

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Two thirds of the cylinder that just holds it.
- ☐ **B** Half of the cylinder that just holds it.
- ☐ **C** All of the cylinder that just holds it.

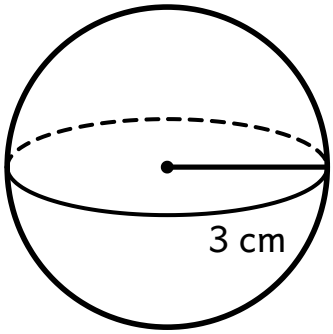
IF THIS FEELS HARD, READ THIS FIRST

- Level 3, Perimeter, area and volume, M3.6.6. If the radius and the area of a circle look new, read this first.
- Level 4, Area, volume and compound units, M4.5.1. If the volume of a cylinder looks new, read this first.
- Level 4, Area, volume and compound units, M4.5.2. If comparing a solid with the cylinder round it looks new, read this first.

BY THE END YOU CAN

- Find the volume and surface area of a sphere
- Find the volume of a half sphere

A football, a marble and the Earth are all close to **spheres**. Every point of a sphere is the same distance from its centre. That distance is the radius r.



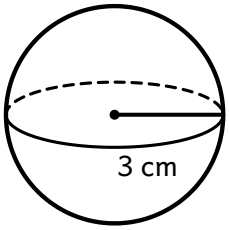
A sphere with radius 3 cm.

A sphere has no flat faces, so we cannot find its volume by slicing it into equal layers. We use a famous fact instead. Put the sphere inside the cylinder that just holds it.

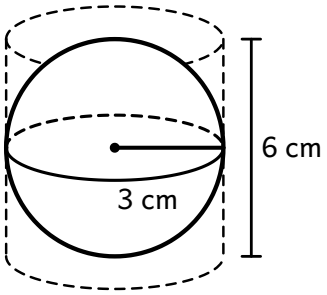
That cylinder has the same radius r. Its height is the width of the sphere, 2r. So its volume is $\pi r^2 \times 2r = 2\pi r^3$.

DRAW IT FIRST

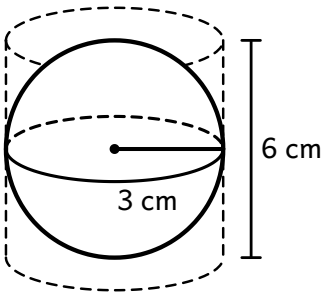
A sphere in *a* cylinder



1 Draw *a* sphere of radius 3 cm.



2 Draw the cylinder that just holds it. It is 3 cm in radius and 6 cm high.



two thirds of the cylinder

3 Archimedes showed that the sphere fills exactly two thirds of this cylinder.

You can test it with water. Fill the cylinder, drop in *a* ball that just fits, and collect the water that spills. The spilled water is two thirds of the cylinder. One third is left in the cylinder.

Two thirds of $2\pi r^3$ is $\frac{2}{3} \times 2\pi r^3 = {}^4/_3 \times \pi r^3$. The volume of *a* sphere is $V = {}^4/_3\ \pi r^3$.

WORKED EXAMPLE

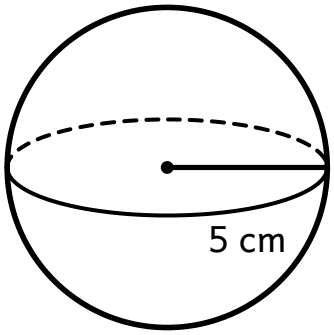
Find the volume of a sphere of radius 3 cm. Give the answer to 1 decimal place.

1. Work out $r^3 = 3 \times 3 \times 3 = 27$.
2. $V = {}^4/_3 \times \pi \times 27$. Now ${}^4/_3$ of 27 is 36, so $V = 36\pi$ cm^3.
3. Put in the π key of a calculator. 36π is 113.1 cm^3, to 1 decimal place.

Answer: 113.1 cm^3

Archimedes also found the surface of the sphere. It has exactly the area of the curved surface of the cylinder that holds it, without the ends. That area is $2\pi r \times 2r = 4\pi r^2$.

Another way to remember it: the surface of a sphere is 4 times the area of a circle with the same radius. So the surface area of a sphere is $A = 4\pi r^2$.



Amina's ball, with radius 5 cm.

WORKED EXAMPLE

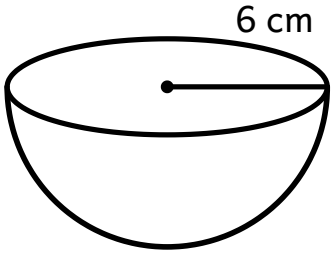
Amina has a ball of radius 5 cm. Find its surface area, to 1 decimal place.

1. The surface area of a sphere is $A = 4\pi r^2$.
2. Work out $r^2 = 5 \times 5 = 25$.
3. $A = 4 \times \pi \times 25 = 100\pi$ cm^2.
4. Put in the π key of a calculator. 100π is 314.2 cm^2, to 1 decimal place.

Answer: 314.2 cm^2

Cut a sphere through its centre and you get two **half spheres**. Each has half the volume, so $V = \frac{1}{2} \times \frac{4}{3} \pi r^3 = \frac{2}{3} \pi r^3$.

The surface of a half sphere needs care. The curved part is half of $4\pi r^2$, which is $2\pi r^2$. A solid half sphere also has a flat face, a circle of area πr^2. With the flat face, the total is $2\pi r^2 + \pi r^2 = 3\pi r^2$.



A half sphere with radius 6 cm.

WORKED EXAMPLE

A bowl is a half sphere of radius 6 cm. Find the volume it can hold, to 1 decimal place.

1. The whole sphere has $V = \frac{4}{3} \times \pi \times 6^3$, and $6^3 = 216$.
2. $\frac{4}{3}$ of 216 is 288, so the whole sphere has volume 288π cm^3.
3. The bowl is half of it, so its volume is $288\pi \div 2 = 144\pi$ cm^3.
4. Put in the π key of a calculator. To 1 decimal place, 144π is 452.4 cm^3.

Answer: 452.4 cm^3

TRY IT

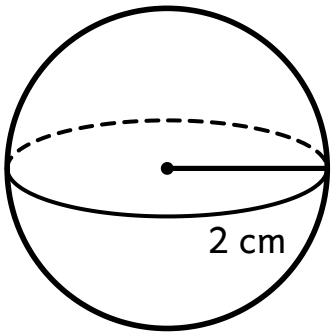
Measure a ball with water

You need: a ball of clay about 4 cm across, a full bowl on a tray, a measuring jug marked in ml.

1. Stand the bowl on the tray. Fill it to the brim.
2. Lower the ball of clay in gently. Water spills onto the tray.
3. Pour the spilled water into the jug and read it.
4. Work out $^4/_3 \times \pi \times 2^3$ for a ball with radius 2 cm, and compare.

What you will see: Each ml of water takes up 1 cm^3. A ball of radius 2 cm has volume about 33.5 cm^3, so about 33 or 34 ml spills.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~If the radius of a sphere doubles, its volume doubles.~~

Why it feels right: Twice the radius looks like twice the size, and a ball twice as wide looks twice as big.

The test that breaks it: The sphere of radius 3 cm has volume 36π, which is about 113.1 cm^3. The sphere of radius 6 cm has volume 288π, which is about 904.8 cm^3. That is 8 times as big, because r is cubed and $2^3 = 8$.

What is true: Volume goes with r^3. Double the radius and the volume is multiplied by 8. The surface area goes with r^2, so it is multiplied by 4.

A WRONG IDEA MANY PEOPLE HAVE

~~The total surface of a solid half sphere is $2\pi r^2$.~~

Why it feels right: A sphere has surface $4\pi r^2$, and half of that is $2\pi r^2$.

The test that breaks it: Press a solid half sphere on a table. The curved part is $2\pi r^2$, but you also see the flat face on the table. That flat circle has area πr^2 and it belongs to the surface of the solid.

What is true: The curved surface is $2\pi r^2$. A solid half sphere has $3\pi r^2$ in all. Always say whether the flat face counts.

KEY WORDS

sphere
a solid ball shape: every point of its surface is the same distance from the centre.

half sphere
half of a sphere, cut through its centre, with a flat circle for a face.

BACK TO YOUR GUESS

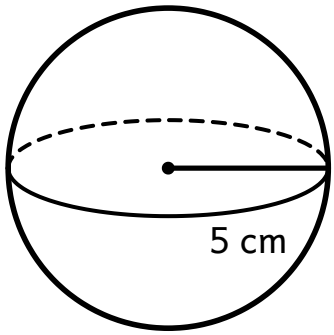
The answer is A: Two thirds of the cylinder that just holds it.

A ball that just fits inside a cylinder fills exactly two thirds of it. The ball touches the sides, the top and the bottom, but it leaves gaps in the corners. This fact was found by Archimedes, over two thousand years ago.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. What is the volume of a sphere of radius 5 cm, to 1 decimal place?



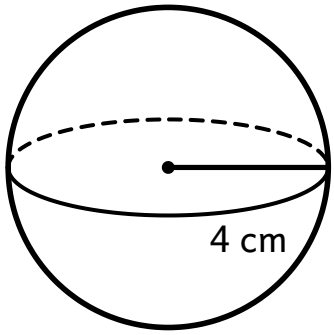
A 314.2 cm^3

B 523.6 cm^3

C 392.7 cm^3

Answer at the back of the book.

2. What is the surface area of a sphere of radius 4 cm, to 1 decimal place?



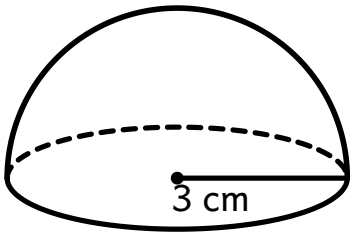
A 268.1 cm^2

B 804.2 cm^2

C 201.1 cm^2

Answer at the back of the book.

3. A solid half sphere has radius 3 cm. What is its volume, to 1 decimal place?



A 56.5 cm^3

B 113.1 cm^3

C 14.1 cm^3

Answer at the back of the book.

4. A solid half sphere has radius 5 cm. Count its flat face. What is its total surface area, to 1 decimal place?

A 157.1 cm^2

B 314.2 cm^2

C 235.6 cm^2

Answer at the back of the book.

5. One sphere has radius 5 cm. A second sphere has radius 10 cm. How many times as big is the volume of the second sphere?

A 8 times as big

B 2 times as big

C 4 times as big

Answer at the back of the book.

6. **From an earlier chapter**

A cone has radius 5 cm and perpendicular height 9 cm. What is its volume, to 1 decimal place?

Answer at the back of the book.

7. A sphere has radius 9 cm. What is its volume, to 1 decimal place?

Answer at the back of the book.

8. A sphere has radius 7 cm. What is its surface area, to 1 decimal place?

   Answer at the back of the book.

9. A solid half sphere has radius 4 cm. What is its volume, to 1 decimal place?

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Sphere: $V = \frac{4}{3}\pi r^3$, $A = 4\pi r^2$. Half sphere: $V = \frac{2}{3}\pi r^3$, curved surface $2\pi r^2$, with the flat face $3\pi r^2$. Radius doubles, volume × 8.

Draw: A sphere inside a cylinder that just holds it, with r and 2r marked.

Joined and cut solids

How much does a flower pot hold, when it is a cone with its tip cut off?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Add the volume of the whole cone and the volume of the tip.
- ☐ **B** Take the volume of the tip away from the volume of the whole cone.
- ☐ **C** Take the radius of the top away from the radius of the base and use the cone formula.

IF THIS FEELS HARD, READ THIS FIRST

- Level 4, Area, volume and compound units, M4.5.2. If the volume of a cone looks new, read this first.
- Level 4, Area, volume and compound units, M4.5.1. If the volume of a cylinder looks new, read this first.
- Level 4, Similarity, transformations and circles, M4.4.5. If working out a missing length in similar shapes looks new, read this first.
- Level 3, Ratio and proportion, M3.4.1. If comparing two lengths as a ratio looks new, read this first.

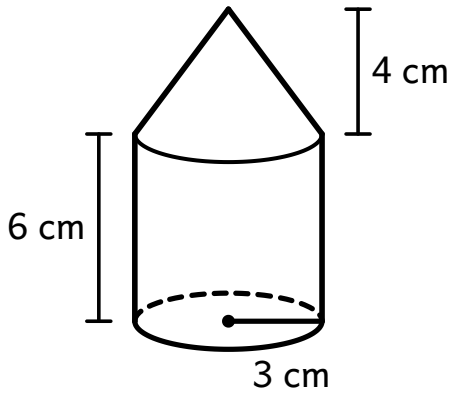
BY THE END YOU CAN

- Find the volume of a cone with its tip cut off
- Find the volume of a solid made of a cylinder and a cone

A sharpened pencil, a rocket and a flower pot are not one of the basic solids. But each one is made of basic solids that you know. The method is always the same.

Split the solid into parts you know, such as a cylinder and a cone. Find the volume of each part. Then **add** the volumes if the parts are joined. **Subtract** if a part has been cut away.

First, joined solids. Here a cylinder with radius 3 cm and height 6 cm has a cone on top. The cone has the same radius, 3 cm, and height 4 cm.



A cylinder and a cone with the same radius.

WORKED EXAMPLE

Find the volume of the solid in the picture. Give the answer to 1 decimal place.

1. The cylinder has volume $\pi \times 3^2 \times 6 = 54\pi$ cm^3.
2. The cone has volume $\frac{1}{3} \times \pi \times 3^2 \times 4 = 12\pi$ cm^3.
3. Add the two parts: $54\pi + 12\pi = 66\pi$ cm^3.
4. To 1 decimal place, 66π is 207.3 cm^3.

Answer: 207.3 cm^3

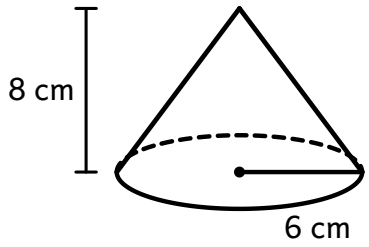
Take care with the heights. The cone has only its own height, 4 cm. Do not use the height of the whole solid, 10 cm, for the cone or for the cylinder.

Now a cut solid. Cut the tip off a cone with a straight cut parallel to the base. The part that is left is a **frustum**. The tip you cut off is a small cone, and the whole cone is a big cone.

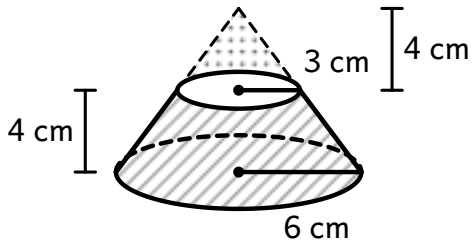
The frustum is the big cone with the small cone taken away. So volume of frustum = volume of big cone − volume of small cone.

DRAW IT FIRST

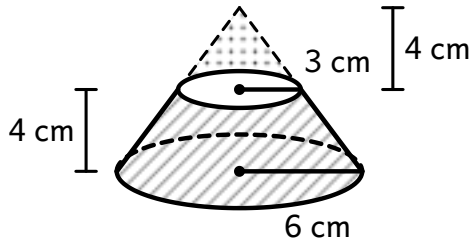
A cone with its tip cut off



1 A cone with base radius 6 cm and height 8 cm.



2 Cut the tip off halfway up. The tip is a small cone with radius 3 cm and height 4 cm.



frustum = big cone − small cone

3 What is left is the frustum: the big cone minus the small cone.

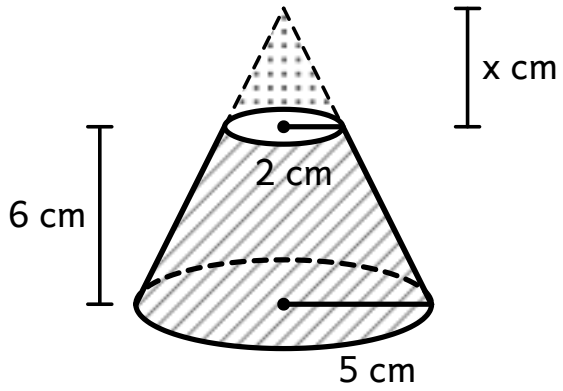
WORKED EXAMPLE

A pot is a frustum. Its base radius is 6 cm, its top radius is 3 cm and its height is 4 cm. The tip that was cut off is a cone with height 4 cm. Find its volume, to 1 decimal place.

1. The big cone has radius 6 cm and height 4 + 4 = 8 cm.
2. Its volume is $\frac{1}{3} \times \pi \times 6^2 \times 8 = 96\pi$ cm^3.
3. The tip is a cone with radius 3 cm and height 4 cm. Its volume is $\frac{1}{3} \times \pi \times 3^2 \times 4 = 12\pi$ cm^3.
4. The frustum is the big cone minus the tip: $96\pi - 12\pi = 84\pi$ cm^3.
5. To 1 decimal place, 84π is 263.9 cm^3.

Answer: 263.9 cm^3

Often you are not told the height of the tip. The small cone has the same shape as the big cone, so they are **similar**. Their lengths are in the same ratio. Use this to find the height of the tip.



A flower pot, with the tip drawn dotted. The height of the tip, x cm, is not given. You find it.

WORKED EXAMPLE

Diego has a flower pot that is a frustum. Its base radius is 5 cm, its top radius is 2 cm and its height is 6 cm. Find its volume, to 1 decimal place.

1. Let the tip have height x cm. The tip and the whole cone are similar, so $x \div (x + 6) = 2 \div 5$.
2. Cross multiply: $5x = 2(x + 6) = 2x + 12$. So $3x = 12$, and $x = 4$.
3. The whole cone has height $4 + 6 = 10$ cm. Radius squared times height is $5^2 \times 10 = 250$ for it, and $2^2 \times 4 = 16$ for the tip.
4. The volume is $\frac{1}{3} \times \pi \times (250 - 16) = \frac{1}{3} \times \pi \times 234 = 78\pi$ cm^3.
5. To 1 decimal place, 78π is 245.0 cm^3.

Answer: 245.0 cm^3

TRY IT

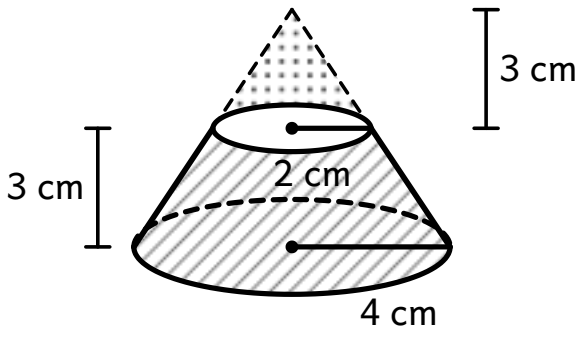
Cut a cone in half

You need: stiff paper, scissors, tape and a ruler.

1. Roll the paper into a cone and tape it. Trim the base flat.
2. Mark the middle of the sloping side, halfway from the base to the apex.
3. Cut straight across at that mark, level with the base.
4. Stand the tip beside the frustum and compare their widths.

What you will see: The tip is a small copy of the whole cone. It is half as wide and half as tall. That is why the tip is similar to the whole cone, and why we can use ratios.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~A frustum is a cylinder with the average of the two radii.~~

Why it feels right: The radius goes from big to small, so the average radius looks like a fair single radius.

The test that breaks it: Take the frustum with radius 6 cm and 3 cm and height 4 cm. The average radius is 4.5 cm. $\pi \times 4.5^2 \times 4 = 81\pi$, which is 254.5 cm^3 to 1 decimal place. The true volume is 84π, which is 263.9 cm^3. The answers differ.

What is true: A frustum is a big cone minus a small cone. Find both volumes and subtract.

A WRONG IDEA MANY PEOPLE HAVE

~~A cylinder with a cone on top has volume $\pi r^2 \times$ the whole height.~~

Why it feels right: The cone sits on the cylinder and has the same radius, so the whole thing looks like one tall tube.

The test that breaks it: Take the joined solid with radius 3 cm, a cylinder of height 6 cm and a cone of height 4 cm. A cylinder of height 10 cm has volume $\pi \times 9 \times 10 = 90\pi$, which is 282.7 cm^3 to 1 decimal place. The true volume is 66π, which is 207.3 cm^3 to 1 decimal place.

What is true: The cone is only one third as big as a cylinder of its height. Find the two parts separately and add.

KEY WORDS

frustum
what is left of a cone when its tip is cut off by a cut parallel to the base.

similar
the same shape but a different size, with all the lengths in the same ratio.

BACK TO YOUR GUESS

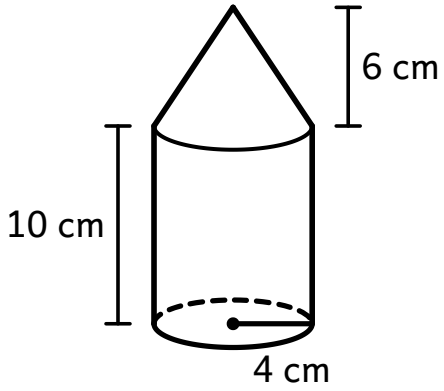
The answer is B: Take the volume of the tip away from the volume of the whole cone.

The pot is what is left when you cut the tip off a whole cone. So its volume is the whole cone minus the tip. Adding would count the tip twice. Taking away radii makes a different cone altogether.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. A solid is a cylinder with a cone on top. Both have radius 4 cm. The cylinder is 10 cm high and the cone is 6 cm high. What is the volume of the whole solid, to 1 decimal place?



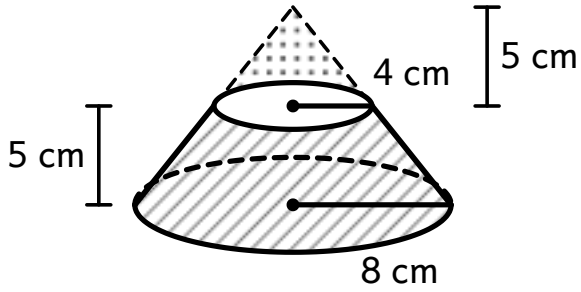
A 603.2 cm^3

B 804.2 cm^3

C 502.7 cm^3

Answer at the back of the book.

2. A pot is a cone with its tip cut off. Its base radius is 8 cm and its top radius is 4 cm. The pot is 5 cm high and the tip is also 5 cm high. What is the volume of the pot, to 1 decimal place?



A 565.5 cm^3

B 670.2 cm^3

C 586.4 cm^3

Answer at the back of the book.

3. A cone has its tip cut off. The base radius is 6 cm, the top radius is 2 cm and the height of the part left is 8 cm. How tall is the tip that was cut off?

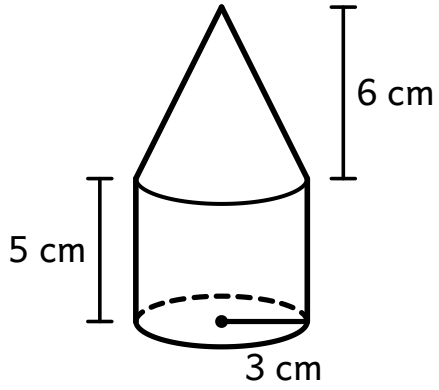
A 2.7 cm

B 4 cm

C 12 cm

Answer at the back of the book.

4. A model rocket is a cylinder with radius 3 cm and height 5 cm. A cone 6 cm high sits on top. What is its volume, to 1 decimal place?



A 197.9 cm^3

B 311.0 cm^3

C 141.4 cm^3

Answer at the back of the book.

5. A cone is cut halfway up its height, parallel to the base. The tip is a cone with every length 2 times smaller than the whole cone. What fraction of the whole cone's volume is the tip?

A $1/2$

B $1/8$

C $1/4$

Answer at the back of the book.

6. **From an earlier chapter**

A sphere has radius 6 cm. What is its volume, to 1 decimal place?

Answer at the back of the book.

7. A solid is a cylinder of radius 4 cm and height 7 cm, with a cone of height 9 cm on top. What is its volume, to 1 decimal place?

Answer at the back of the book.

8. A pot has base radius 6 cm, top radius 3 cm and height 5 cm. It is a cone with its tip cut off. What is its volume, to 1 decimal place?

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Joined solid: add the parts. Cut solid: subtract the part that is gone. Frustum = big cone − small cone. Similar triangles give the missing height.

Draw: A frustum with its tip dotted above it, and the radii and the two heights marked.

M4.5.5 CHAPTER 5

Changing units of area and volume

How many square centimetres are in one square metre?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** 100 square centimetres.
- ☐ **B** 1 000 square centimetres.
- ☐ **C** 10 000 square centimetres.

IF THIS FEELS HARD, READ THIS FIRST

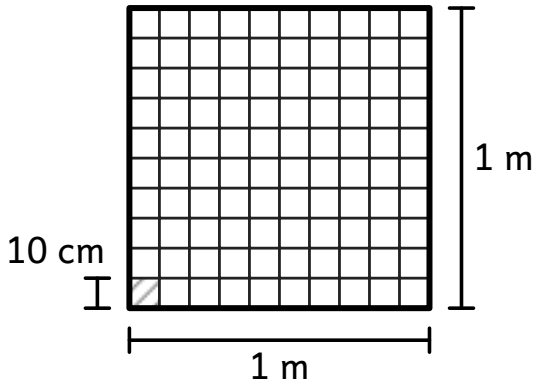
- Level 3, Perimeter, area and volume, M3.6.1. If changing metres to centimetres looks new, read this first.
- Level 3, Perimeter, area and volume, M3.6.3. If area as length × width looks new, read this first.
- Level 3, Perimeter, area and volume, M3.6.7. If volume as length × width × height looks new, read this first.

BY THE END YOU CAN

- Change between cm^2 and m^2, and between cm^3 and m^3
- Change between cm^3, litres and m^3

You know that 1 m = 100 cm. It is tempting to think that 1 m^2 = 100 cm^2 as well. It is not. This chapter shows why, and gives one rule that works for every unit.

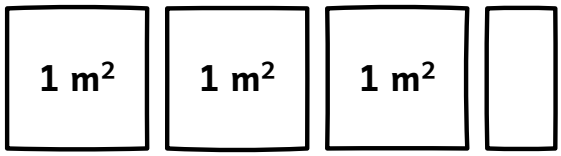
A square metre is the area of a square 1 m long and 1 m wide. We can cut that square into smaller squares, 10 cm by 10 cm. The picture shows them.



One square metre, cut into 100 squares of 10 cm by 10 cm.

Each small square is 10 cm by 10 cm, so its area is 10 × 10 = 100 cm^2. There are 10 rows of 10, which is 100 small squares. So the whole area is 100 × 100 = 10 000 cm^2.

The same idea works for any unit. The square has 100 cm along each edge, and the area is 100 × 100. So 1 m^2 = 10 000 cm^2. Likewise 1 cm = 10 mm, so 1 cm^2 = 10 × 10 = 100 mm^2.



½ m^2 is the narrow rectangle

each 1 m^2 is 10 000 cm^2

The area 3.5 m^2 is three whole square metres and half of another.

WORKED EXAMPLE

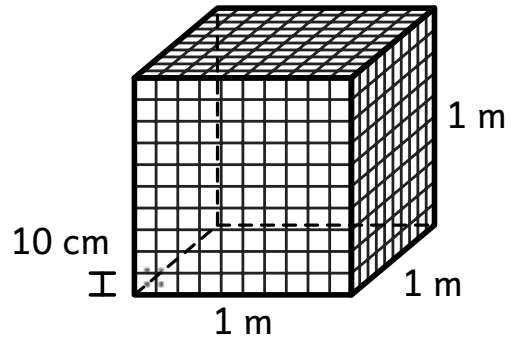
Change 3.5 m^2 into cm^2.

1. One square metre is 10 000 cm^2.
2. There are 3.5 square metres, so multiply: 3.5 × 10 000.
3. 3.5 × 10 000 = 35 000 cm^2.

Answer: 35 000 cm^2

To change cm^2 into m^2, go the other way and divide by 10 000. A small unit gives a big number, and a big unit gives a small number. So 20 000 cm^2 is 2 m^2.

Now volume. A cube 1 m on each edge has volume 1 m^3. It is also 100 cm by 100 cm by 100 cm. The picture cuts it into small cubes, 10 cm on each edge.



One cubic metre. It holds 10 × 10 × 10 = 1 000 small cubes.

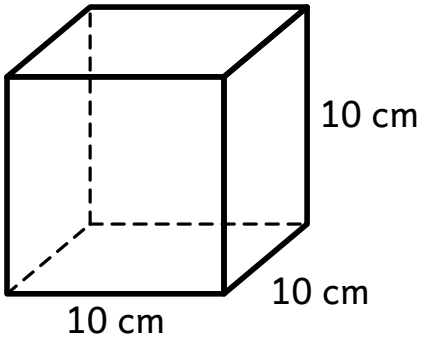
A cube with 100 cm along each edge has volume 100 × 100 × 100 = 1 000 000 cm^3. So 1 m^3 = 1 000 000 cm^3. A million small cubes, each 1 cm on an edge, fill a cubic metre.

Length ×	Area ×	Volume ×
10	100	1 000
100	10 000	1 000 000

When lengths change by a factor, areas change by that factor squared and volumes by that factor cubed.

The table gives the rule. Change the unit of length by a factor k. The unit of area changes by k × k. The unit of volume changes by k × k × k. For m to cm, k = 100.

A **litre** is a unit for liquids. One litre is the volume of a cube 10 cm on each edge. That is 10 × 10 × 10 = 1 000 cm^3. One **millilitre** (ml) is one thousandth of a litre, so 1 ml = 1 cm^3.



1 litre = 1 000 cm^3

One litre is a cube 10 cm on each edge.

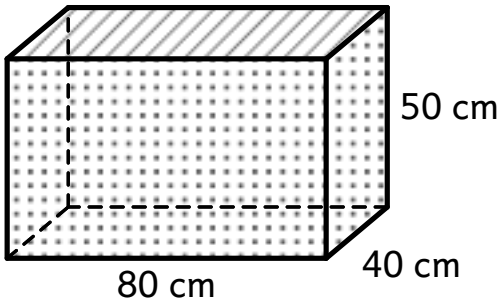
The cubic metre picture held 1 000 small cubes of 10 cm. Each is one litre. So 1 m^3 = 1 000 litres. It also agrees with 1 000 000 cm^3 ÷ 1 000 cm^3 = 1 000.

WORKED EXAMPLE

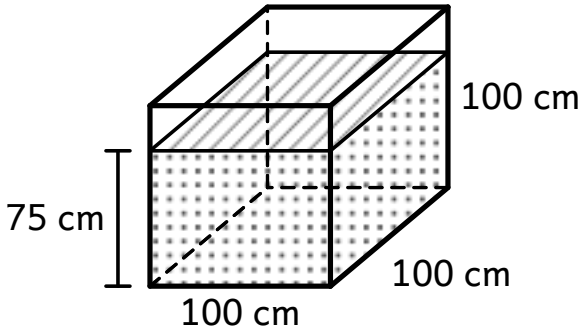
Lucía has a fish tank that is 80 cm long, 40 cm wide and 50 cm high. How many litres does it hold when it is full?

1. The volume is length × width × height = 80 × 40 × 50.
2. 80 × 40 × 50 = 160 000 cm^3.
3. One litre is 1 000 cm^3, so divide by 1 000.
4. 160 000 ÷ 1 000 = 160 litres.

Answer: 160 litres



The fish tank, full to the top.



A water store 1 m by 1 m by 1 m, filled to 75 cm. It holds 0.75 m^3.

WORKED EXAMPLE

A water store holds 0.75 m^3. How many cm^3 is that, and how many litres?

1. One cubic metre is 1 000 000 cm^3.
2. So 0.75 m^3 is 0.75 × 1 000 000 = 750 000 cm^3.
3. One litre is 1 000 cm^3. So 750 000 cm^3 is 750 000 ÷ 1 000 = 750 litres.
4. Check with 1 m^3 = 1 000 litres: 0.75 × 1 000 = 750 litres. The two ways agree.

Answer: 750 000 cm^3 and 750 litres

TRY IT

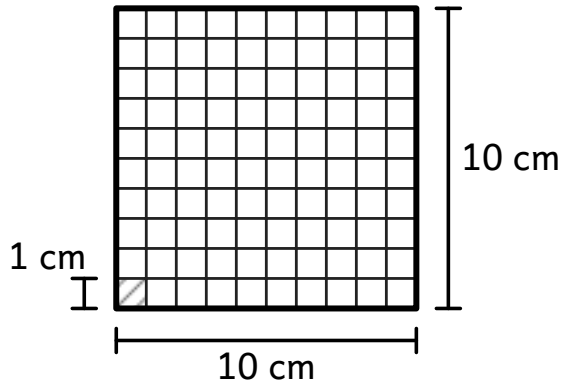
A hundred small squares

You need: squared paper with 1 cm squares, a pencil and a ruler.

1. Draw a square 10 cm by 10 cm on the paper.
2. Count the squares inside it. Count the rows, then the columns.
3. Work out 10 × 10 and compare with your count.
4. Imagine 100 rows of 100 of these small squares. That is one square metre.

What you will see: The square holds 100 squares of 1 cm^2, so its area is 100 cm^2. A square 10 times as long on each side, 100 cm, would hold 10 × 10 times as many, which is 10 000.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~Since 1 m = 100 cm, it follows that 1 m^2 = 100 cm^2.~~

Why it feels right: We change metres to centimetres by multiplying by 100, and the little 2 only seems to say that it is an area.

The test that breaks it: Draw a square 1 m by 1 m. In centimetres it is 100 cm by 100 cm, and its area is 100 × 100 = 10 000 cm^2. It is not 100 cm^2. A 10 cm by 10 cm square has area 100 cm^2, which is a tiny part of a square metre.

What is true: Areas change by the square of the length factor, and volumes by the cube. So 1 m^2 = 10 000 cm^2 and 1 m^3 = 1 000 000 cm^3.

A WRONG IDEA MANY PEOPLE HAVE

~~A litre is 100 cm³, because a litre box is 10 cm long.~~

Why it feels right: The edge of the litre cube is 10 cm, and 10 × 10 = 100 is a number that is quick to remember.

The test that breaks it: A litre cube has three edges of 10 cm. Volume is length × width × height, so 10 × 10 × 10 = 1 000 cm³. A square of 10 × 10 is only a flat face of the cube.

What is true: One litre is 1 000 cm³, so 1 ml is 1 cm³. And 1 000 litres make 1 m³.

KEY WORDS

litre
: a unit for liquids: the volume of a cube 10 cm on each edge, which is 1 000 cm³.

millilitre
: one thousandth of a litre. 1 ml is the same as 1 cm³.

BACK TO YOUR GUESS

The answer is C: 10 000 square centimetres.

A square metre is 100 cm long and 100 cm wide, so it holds 100 × 100 = 10 000 square centimetres. The number is not 100, because 100 is the number of centimetres along one edge.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Change 2.5 m^2 into cm^2.

   A 25 000 cm^2

   B 250 cm^2

   C 2 500 cm^2

   Answer at the back of the book.

2. A piece of cloth has an area of 36 000 cm^2. What is its area in m^2?

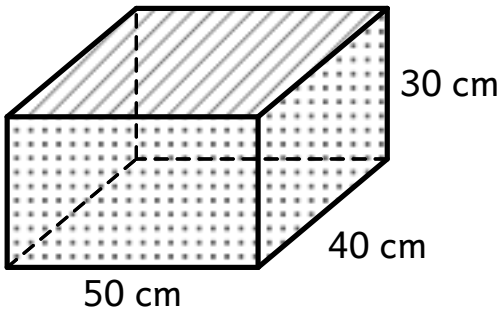
   A 360 m^2

   B 3.6 m^2

   C 36 m^2

   Answer at the back of the book.

3. A tank is 50 cm long, 40 cm wide and 30 cm high. How many litres does it hold when it is full?



   A 60 000 litres

   B 600 litres

   C 60 litres

   Answer at the back of the book.

4. A truck holds 4 m^3 of sand. How many cm^3 is that?

   A 400 000 cm^3

   B 4 000 000 cm^3

   C 4 000 cm^3

   Answer at the back of the book.

5. A rain tank holds 2 400 litres. How many m^3 is that?

   A 2.4 m^3

   B 24 m^3

   C 240 m^3

   Answer at the back of the book.

6. **From an earlier chapter**

   A solid is a cylinder of radius 3 cm and height 4 cm, with a cone 6 cm high on top. What is its volume, to 1 decimal place?

   Answer at the back of the book.

7. Change 0.8 m^2 into cm^2.

   Answer at the back of the book.

8. A tank is 60 cm long, 30 cm wide and 50 cm high. How many litres does it hold when it is full?

   Answer at the back of the book.

9. Change 3 500 000 cm^3 into m^3.

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

1 m^2 = 10 000 cm^2. 1 m^3 = 1 000 000 cm^3. 1 litre = 1 000 cm^3. 1 ml = 1 cm^3. 1 m^3 = 1 000 litres. Small unit to big unit: divide.

Draw: A square metre cut into 100 squares of 10 cm, with one square shaded and labelled 100 cm^2.

M4.5.6 CHAPTER 6

Speed, distance and time

> **A car drives 60 km at 60 km per hour, then 40 km at 20 km per hour. What is its average speed?**

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** About 33 km per hour.
- ☐ **B** 40 km per hour, halfway between 60 and 20.
- ☐ **C** 80 km per hour, the two speeds added.

IF THIS FEELS HARD, READ THIS FIRST

- Level 3, Ratio and proportion, M3.4.6. If working out a speed in km per hour from distance and time looks new, read this first.
- Level 3, Speed, forces and pressure, S3.7.1. If speed in metres per second looks new, read this first.
- Level 3, Coordinates and straight lines, M3.9.8. If a distance time graph looks new, read this first.

BY THE END YOU CAN

- Use speed = distance ÷ time with the right units
- Change km per hour into metres per second
- Find the average speed for a journey in two parts

Speed tells you how far something goes in one unit of time. A cyclist who goes 15 km in each hour has a speed of 15 km per hour, written 15 km/h.

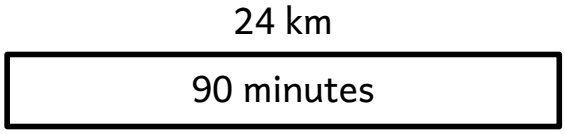
Km per hour is a **compound unit**. It is made of two units, a unit of distance (km) and a unit of time (hours), joined by "per". Metres per second (m/s) is another.

The rule is speed = distance ÷ time. It can be turned round: distance = speed × time, and time = distance ÷ speed.

To find	Use
speed	distance ÷ time
distance	speed × time
time	distance ÷ speed

One rule, three ways to use it.

The units must fit. If the distance is in km and the time is in hours, the speed is in km/h. If the time is given in minutes, change it to hours first. There are 60 minutes in an hour.



A cyclist rides 24 km in 90 minutes.

WORKED EXAMPLE

Ravi rides his bicycle 24 km in 90 minutes. What is his speed in km/h?

1. The time is in minutes. Change it to hours: 90 ÷ 60 = 1.5 hours.
2. Speed = distance ÷ time = 24 ÷ 1.5.
3. 24 ÷ 1.5 = 16 km/h.

Answer: 16 km/h

Now change units. In science, speeds are often in metres per second. One km/h is 1 000 m in 3 600 seconds, which is 1 ÷ 3.6 m/s. So to change km/h into m/s, divide by 3.6. To change m/s into km/h, multiply by 3.6.

km/h	m/s
36	10
72	20
108	30

Divide the number in km/h by 3.6 to get m/s.

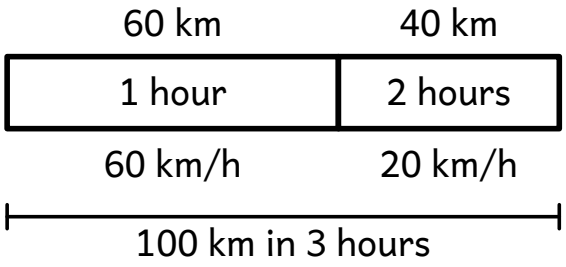
WORKED EXAMPLE

A train goes at 72 km/h. What is its speed in m/s?

1. One hour is 3 600 seconds, and 1 km is 1 000 m.
2. So 1 km/h = 1 000 ÷ 3 600 m/s, which is 1 ÷ 3.6 m/s.
3. To change km/h to m/s, divide by 3.6.
4. 72 ÷ 3.6 = 20 m/s.

Answer: 20 m/s

Now a journey in two parts. A car drives 60 km in 1 hour, then 40 km in the next 2 hours. The bar below shows the two parts. Each part is as long as its distance.



A journey in two parts. The speeds are 60 km/h and 20 km/h.

It is tempting to say the average speed is halfway between 60 and 20, which is 40 km/h. That is wrong, because the car spends 2 hours at 20 km/h and only 1 hour at 60 km/h.

The ***average speed*** is the one steady speed that covers the same total distance in the same total time. So average speed = total distance ÷ total time.

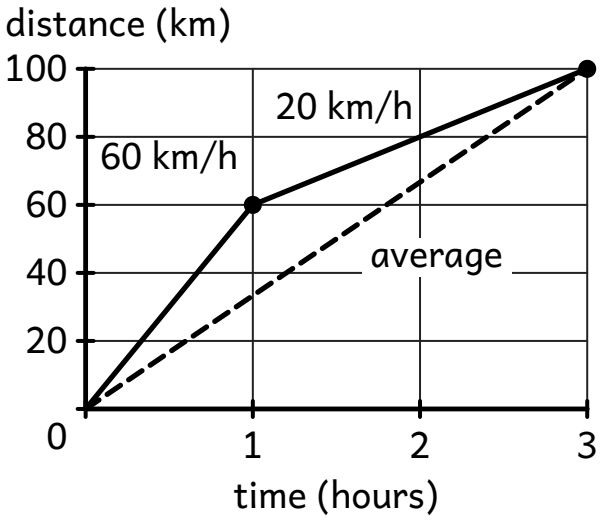
WORKED EXAMPLE

A car drives 60 km in 1 hour, then 40 km in 2 hours. What is its average speed, to 1 decimal place?

1. Total distance = 60 + 40 = 100 km.
2. Total time = 1 + 2 = 3 hours.
3. Average speed = total distance ÷ total time = 100 ÷ 3.
4. To 1 decimal place, 100 ÷ 3 = 33.3 km/h.
5. This is not the mean of the two speeds, 60 and 20, which would be 40.

Answer: 33.3 km/h

A distance time graph shows the same journey. Time goes across and distance goes up. The steeper the line, the faster the speed. The dashed line is the average, a steady journey from the start to the end.



The dashed line is steady. It reaches the same end point in the same time.

TRY IT

Measure your walking speed

You need: a long tape or string, marked at 20 m, a clock or phone that shows seconds, a friend to time you.

1. Lay the string flat along a straight path, 20 m long.
2. Walk along it at your usual pace. Your friend times you in seconds.
3. Work out speed = distance ÷ time, in m/s.
4. Multiply by 3.6 to change it to km/h.

What you will see: Most people cover 1 to 1.5 m every second when they walk. That is about 3.6 to 5.4 km/h.

If you cannot try it, this is what happens.

20 m

15 seconds

about 1.3 m/s

A WRONG IDEA MANY PEOPLE HAVE

~~The average speed of a journey is the mean of the two speeds.~~

Why it feels right: We find an average of numbers by adding them and dividing, so it looks right to do it with speeds.

The test that breaks it: Drive 60 km at 60 km/h, then 40 km at 20 km/h. The mean of 60 and 20 is 40 km/h. But the car goes 100 km in 3 hours, which is 33.3 km/h. The car spent longer at the slow speed.

What is true: Average speed is the total distance divided by the total time. Use the mean of two speeds only if the times are equal.

A WRONG IDEA MANY PEOPLE HAVE

~~A cyclist who rides 24 km in 90 minutes has a speed of 24 ÷ 90 km/h.~~

Why it feels right: The rule says divide the distance by the time, and the time is the number 90.

The test that breaks it: 24 ÷ 90 = 0.27, which is far slower than anyone walks. But km/h needs the time in hours. 90 minutes is 1.5 hours, and 24 ÷ 1.5 = 16 km/h.

What is true: Make the units fit first. For km/h the time must be in hours.

KEY WORDS

compound unit
a unit made of two units, such as km per hour or metres per second.

average speed
the total distance of a journey divided by the total time.

BACK TO YOUR GUESS

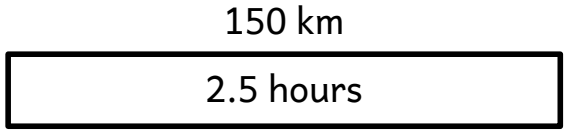
The answer is A: About 33 km per hour.

The average speed is the total distance divided by the total time. The car goes 100 km in 3 hours, which is 33.3 km per hour. It spends longer at the slow speed, so the average is pulled towards 20, not halfway.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. A train goes 150 km in 2.5 hours at a steady speed. What is its speed?



   A 60 km/h

   B 375 km/h

   C 152.5 km/h

   Answer at the back of the book.

2. A bus goes at 45 km/h for 40 minutes. How far does it go?

   A 1 800 km

   B 1.125 km

   C 30 km

   Answer at the back of the book.

3. A lorry must go 210 km at 60 km/h. How long does the journey take?

   A 3 hours 50 minutes

   B 0.29 hours

   C 12 600 hours

   D 3.5 hours

   Answer at the back of the book.

4. A car goes at 90 km/h. What is its speed in m/s?

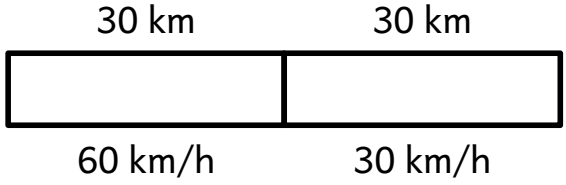
   A 25 m/s

   B 324 m/s

   C 1.5 m/s

   Answer at the back of the book.

5. A car drives 30 km at 60 km/h. Then it drives 30 km at 30 km/h. What is its average speed for the whole journey?



A 45 km/h

B 60 km/h

C 40 km/h

Answer at the back of the book.

6. **From an earlier chapter**

Change 4.2 m^2 into cm^2.

Answer at the back of the book.

7. A cyclist rides 18 km in 45 minutes. What is the speed in km/h?

Answer at the back of the book.

8. A train goes at 54 km/h. What is its speed in m/s?

Answer at the back of the book.

9. A car drives 80 km at 40 km/h, then 60 km at 60 km/h. What is its average speed, to 1 decimal place?

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Speed = distance ÷ time. Distance = speed × time. Time = distance ÷ speed. km/h to m/s: ÷ 3.6. Average speed = total distance ÷ total time.

Draw: A bar in two parts with its distances and times, and the total under a bracket.

M4.5.7 CHAPTER 7

Density and pressure

Why does a big block of wood float, while a small iron nail sinks?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Heavy things sink and light things float.
- ☐ **B** Big things always float and small things always sink.
- ☐ **C** A thing floats when it has less mass in each cm^3 than water has.

IF THIS FEELS HARD, READ THIS FIRST

- Level 3, Particles, S3.5.8. If density as mass packed into a space looks new, read this first.
- Level 3, Speed, forces and pressure, S3.7.7. If pressure as force on an area looks new, read this first.
- Level 4, Area, volume and compound units, M4.5.5. If changing cm^2 into m^2 looks new, read this first.

BY THE END YOU CAN

- Use density = mass ÷ volume to find any one of the three
- Use pressure = force ÷ area
- Find the mass of a block from its size and density

Two blocks of the same size can have very different masses. A block of iron is much heavier than a block of wood of the same size. The iron has more mass packed into each cm^3.

Density says how much mass there is in each unit of volume. Density = mass ÷ volume. Its unit is a compound unit: grams per cm^3, written g/cm^3. Another is kg

per m^3.

Water has a density of about 1 g/cm^3. So 1 cm^3 of water has a mass of about 1 g. A litre of water, which is 1 000 cm^3, has a mass of about 1 kg.

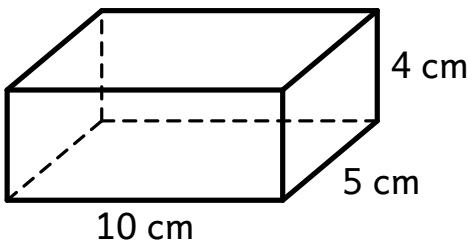
Material	About g/cm^3
Oak wood	0.7
Ice	0.92
Water	1
Aluminium	2.7
Iron	7.9
Gold	19.3

Some densities. A solid object made of one material floats in water if its density is less than 1 g/cm^3.

The rule density = mass ÷ volume can be turned round, in the same way as speed. Use the table to choose.

To find	Use
density	mass ÷ volume
mass	density × volume
volume	mass ÷ density

One rule, three ways to use it.



mass 540 g

Yusuf's block of metal: 10 cm by 5 cm by 4 cm, with a mass of 540 g.

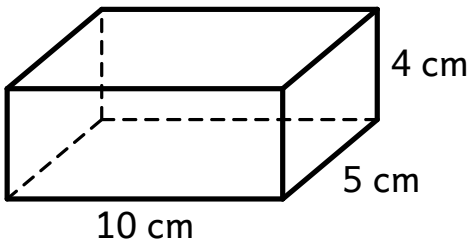
WORKED EXAMPLE

Yusuf has a block of metal with a mass of 540 g and a volume of 200 cm³. What is its density?

1. Density = mass ÷ volume.
2. Density = 540 ÷ 200 g/cm³.
3. 540 ÷ 200 = 2.7 g/cm³. That matches aluminium in the table.

Answer: 2.7 g/cm³

To find a mass from a size, first find the volume of the block. Then multiply the volume by the density. The mass is in grams if the volume is in cm³ and the density is in g/cm³.



A block of iron, 10 cm by 5 cm by 4 cm.

WORKED EXAMPLE

Iron has a density of about 7.9 g/cm³. Find the mass of an iron block 10 cm by 5 cm by 4 cm.

1. The volume of the block is 10 × 5 × 4 = 200 cm³.
2. Mass = density × volume = 7.9 × 200.
3. 7.9 × 200 = 1 580 g.
4. That is 1.58 kg, because 1 000 g is 1 kg.

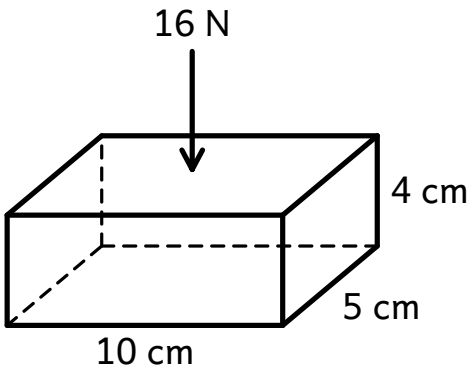
Answer: About 1 580 g

Now pressure. A force is a push or a pull, measured in newtons (N). **Pressure** is how hard a force pushes on each unit of area. Pressure = force ÷ area.

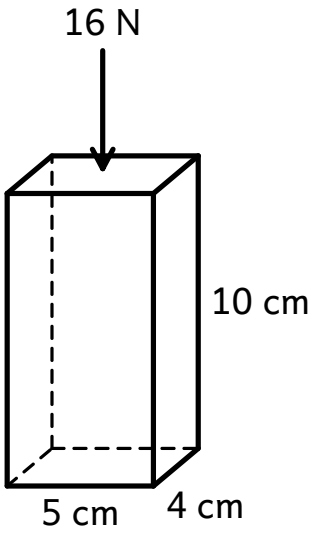
Pressure is measured in newtons per cm^2 (N/cm^2), or in newtons per m^2 (N/m^2). One newton per m^2 is one **pascal** (Pa).

The same force can give a small or a big pressure. A block lying on its large face spreads its weight over more area than the same block standing on its small face.

Weight is a force, the pull of the Earth on something. Every 1 kg of mass weighs about 10 N. The iron block has a mass of 1.58 kg, so it weighs about 16 N.



The block lies on its bottom face, 10 cm by 5 cm, which is 50 cm^2.



The block stands on its bottom face, 5 cm by 4 cm, which is 20 cm^2.

WORKED EXAMPLE

A block weighing about 16 N stands on a face of 5 cm by 4 cm. Find the pressure on the table, first in N/cm² and then in pascals.

1. The area of the face is 5 × 4 = 20 cm².
2. Pressure = force ÷ area = 16 ÷ 20 = 0.8 N/cm².
3. One m² is 10 000 cm², so 1 N/cm² is the same as 10 000 N/m².
4. So 0.8 N/cm² is 0.8 × 10 000 = 8 000 N/m², which is about 8 000 pascals.

Answer: About 0.8 N/cm², or about 8 000 pascals

TRY IT

Sink or float

You need: a bowl of water, a stone, a coin, a piece of wood and a candle.

1. Guess for each thing: will it sink or float?
2. Put the things in the water one at a time.
3. Write down what happened.

What you will see: The stone and the coin sink, and the wood and the candle float. It does not depend on how big a thing is. It depends on its density compared with water.

If you cannot try it, this is what happens.

Thing	What happens
Stone	sinks
Coin	sinks
Wood	floats
Candle	floats

A WRONG IDEA MANY PEOPLE HAVE

~~Heavy things sink and light things float.~~

Why it feels right: Most heavy things we pick up, like stones, do sink, and a feather floats.

The test that breaks it: A big log has a mass of many kilograms, and it floats. A small steel nail has a mass of a few grams, and it sinks. Mass alone does not decide it.

What is true: A solid object made of one material sinks in water if its density is more than 1 g/cm^3. It floats if its density is less. Compare the density, not the mass.

A WRONG IDEA MANY PEOPLE HAVE

~~Pressure is the same as force.~~

Why it feels right: A big force feels like a big pressure when you push hard with your hand.

The test that breaks it: The block weighs about 16 N in both pictures. Lying down, the pressure is 16 ÷ 50 = 0.32 N/cm^2. Standing, it is 16 ÷ 20 = 0.8 N/cm^2. The force is the same, but the pressure is 2.5 times as big.

What is true: Pressure is force divided by area. The same force on a smaller area gives a bigger pressure. That is why a sharp knife cuts.

KEY WORDS

density
the mass in each unit of volume. Density = mass ÷ volume.

pressure
the force on each unit of area. Pressure = force ÷ area.

pascal
the unit of pressure. One pascal is one newton per m^2.

BACK TO YOUR GUESS

The answer is C: A thing floats when it has less mass in each cm^3 than water has.

A big log is heavy, but each cm^3 of wood has less mass than each cm^3 of water. So it floats. A small nail is light, but each cm^3 of iron has much more mass than water. So it sinks. What counts is the mass in each cm^3, which is called density.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. A stone has a mass of 150 g and a volume of 60 cm^3. What is its density?

   A 9 000 g/cm^3

   B 2.5 g/cm^3

   C 0.4 g/cm^3

   Answer at the back of the book.

2. Aluminium has a density of 2.7 g/cm^3. A block of aluminium is 10 cm by 8 cm by 5 cm. What is its mass?

   A 1 080 g

   B 148.1 g

   C 216 g

   Answer at the back of the book.

3. A metal has a density of 8 g/cm^3. A piece of it has a mass of 480 g. What is its volume?

   A 3 840 cm^3

   B 0.017 cm^3

   C 60 cm^3

   Answer at the back of the book.

4. A force of 600 N pushes on an area of 0.05 m^2. What is the pressure?

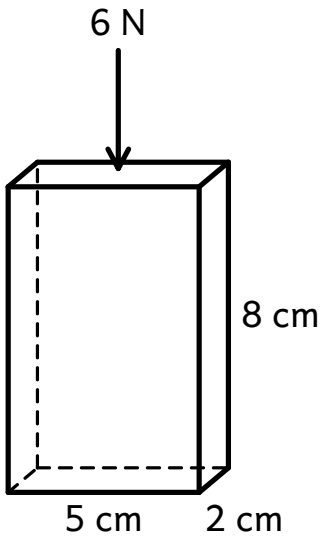
   A 30 Pa

   B 12 000 Pa

   C 0.000083 Pa

   Answer at the back of the book.

5. An iron block is 8 cm tall and stands on a face of 5 cm by 2 cm. It weighs about 6 N. What is the pressure on the table?



   A 60 N/cm^2

   B 0.15 N/cm^2

   C About 0.6 N/cm^2

   Answer at the back of the book.

6. **From an earlier chapter**

   A train goes at 126 km/h. What is its speed in m/s?

   Answer at the back of the book.

7. A block has a mass of 480 g and a volume of 160 cm^3. What is its density, in g/cm^3?

   Answer at the back of the book.

8. A block of a material with density 2.5 g/cm^3 is 4 cm by 5 cm by 6 cm. What is its mass, in g?

   Answer at the back of the book.

9. A force of 90 N pushes on an area of 30 cm^2. What is the pressure, in N/cm^2?

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Density = mass ÷ volume. Water is about 1 g/cm^3. Pressure = force ÷ area. N/m^2 is the pascal. 1 N/cm^2 = 10 000 N/m^2. Smaller area, bigger pressure.

Draw: A block standing on its small face, and the same block lying flat. Put a 16 N arrow on each.

M4.5.8 CHAPTER 8

Rates of flow and rate graphs

> **A tap fills a tank at 12 litres a minute. The tank holds 120 litres. How long does it take?**

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** 10 minutes, because 120 ÷ 12 = 10.
- ☐ **B** 108 minutes, because 120 − 12 = 108.
- ☐ **C** 1 440 minutes, because 120 × 12 = 1 440.

IF THIS FEELS HARD, READ THIS FIRST

- Level 4, Area, volume and compound units, M4.5.5. If changing cm^3 into litres looks new, read this first.
- Level 4, Area, volume and compound units, M4.5.6. If speed as distance ÷ time looks new, read this first.
- Level 3, Coordinates and straight lines, M3.9.7. If the gradient of a straight line looks new, read this first.

BY THE END YOU CAN

- Find how long a tank takes to fill at 12 litres a minute
- Read the gradient of a straight line graph as a rate

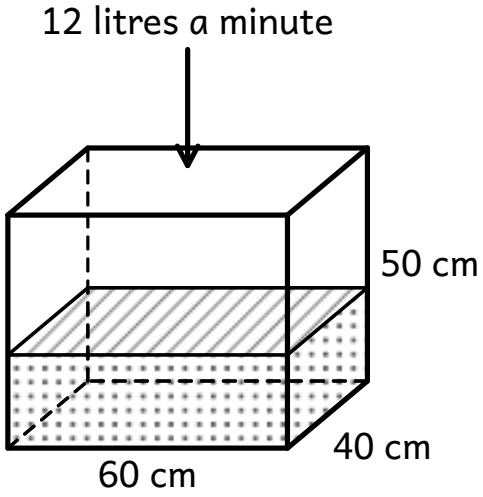
A **rate of flow** says how much liquid moves in each unit of time. A tap that gives 12 litres every minute has a rate of 12 litres per minute, written 12 litres/min. It is a compound unit, like km/h.

The rule is the same shape as the rule for speed. Rate = volume ÷ time. It can be turned round: volume = rate × time, and time = volume ÷ rate.

To find	Use
rate	volume ÷ time
volume	rate × time
time	volume ÷ rate

One rule, three ways to use it.

The picture shows a tank with a tap above it. The tank is partly full, and more water is flowing in. The tank is 60 cm long, 40 cm wide and 50 cm deep.



A tank that fills at 12 litres a minute.

WORKED EXAMPLE

Hana has the tank in the picture. It is 60 cm long, 40 cm wide and 50 cm deep. It starts empty, and the picture shows it part way through filling. A tap fills it at 12 litres per minute. How long does it take to fill the whole tank?

1. The volume of the tank is $60 \times 40 \times 50 = 120\,000$ cm^3.
2. Change to litres. One litre is 1 000 cm^3, so $120\,000 \div 1\,000 = 120$ litres.
3. Time = volume ÷ rate = 120 ÷ 12.
4. 120 ÷ 12 = 10 minutes.

Answer: 10 minutes

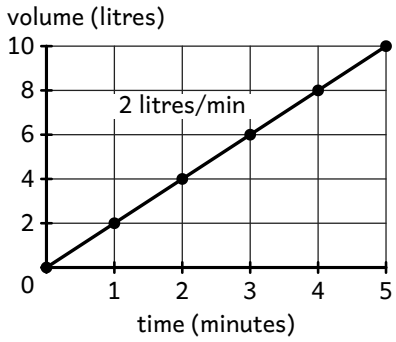
Now draw a graph. Put time in minutes across, and volume in litres up. When the tap runs steadily, each minute adds the same volume. So the points lie on a straight line through (0, 0).

SEE IT THREE WAYS

With things

A tap fills a jug with 2 litres every minute. Pour each jugful into a tank and write down how much water the tank holds after each minute.

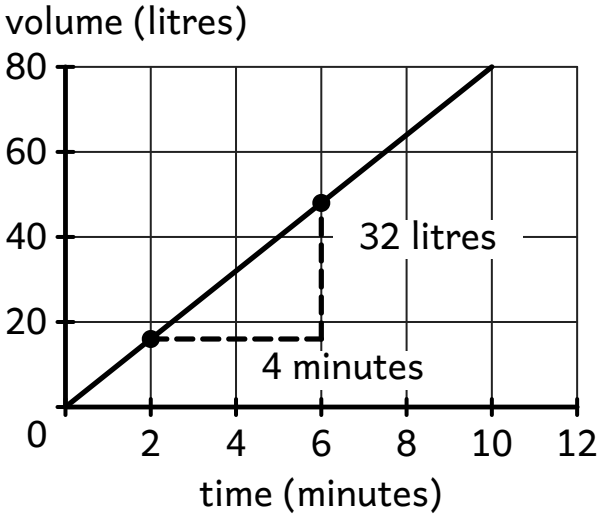
As a picture



In symbols

Volume = rate × time. After 5 minutes the tank holds 2 × 5 = 10 litres.

The steepness of the line is its ***gradient***. The gradient is how much the volume goes up for each 1 minute across. That is the rate of flow. Gradient = change in volume ÷ change in time.



Two points on the line. Across 4 minutes the volume goes up 32 litres.

WORKED EXAMPLE

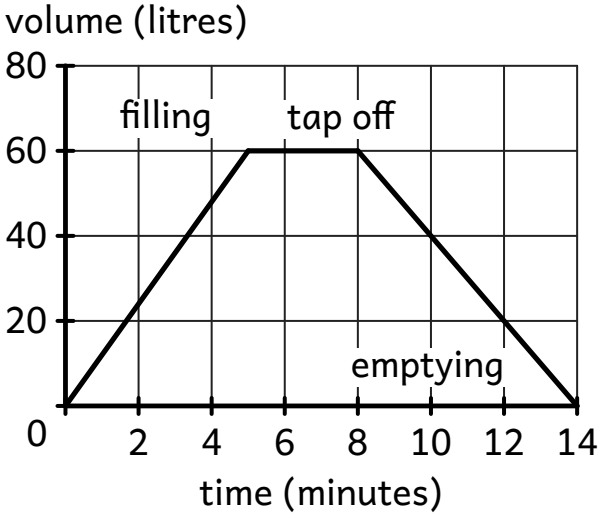
The graph shows water running steadily into a tank. The line goes through (2, 16) and (6, 48). Find the rate of flow.

1. Choose two points on the line that are far apart: (2, 16) and (6, 48).
2. The volume goes up by 48 − 16 = 32 litres.
3. The time goes up by 6 − 2 = 4 minutes.
4. Gradient = 32 ÷ 4 = 8 litres per minute.

Answer: 8 litres per minute

A graph can have more than one part. A flat part means that the volume is not changing, so the rate is 0. A line that goes down means water is leaving the tank.

Hana's tank holds 120 litres. The graph shows it filled, left, and then emptied.



A tank fills, rests, and empties.

WORKED EXAMPLE

The graph shows Hana's tank, which holds 120 litres. It is filled, left, and then emptied. Find the rate of flow when it fills and when it empties.

1. Filling: the tank has 60 litres after 5 minutes. The rate is 60 ÷ 5 = 12 litres per minute.
2. The flat part, from 5 to 8 minutes, has a gradient of 0. No water flows.
3. Emptying: the water goes from 60 litres to 0 between 8 and 14 minutes. That takes 14 − 8 = 6 minutes.
4. Water leaves at 60 ÷ 6 = 10 litres per minute.

Answer: 12 litres per minute in, 10 litres per minute out

TRY IT

Time a tap

You need: a tap, a 1 litre bottle or jug, a clock that shows seconds.

1. Open the tap fully and hold the bottle under it.
2. Time how many seconds it takes to fill the bottle to the 1 litre mark.
3. Work out how many litres would flow in 60 seconds: 60 ÷ your time.

What you will see: The table shows the seconds taken to fill 1 litre, and the rate in litres per minute. If the bottle fills in 10 seconds, the tap gives 6 litres in a minute. If it fills in 5 seconds, it gives 12 litres a minute.

If you cannot try it, this is what happens.

Seconds	Litres/min
10	6
5	12

A WRONG IDEA MANY PEOPLE HAVE

~~**A flat part of a volume time graph means that the tank is full.**~~

Why it feels right: The line stops going up, and a full tank cannot take any more water.

The test that breaks it: Look at the graph of the tank. The line is flat at 60 litres from 5 to 8 minutes, but the tank holds 120 litres. It is half full, and the tap is turned off.

What is true: A flat part means the volume is not changing. The rate is 0. It does not tell you whether the tank is full or not.

A WRONG IDEA MANY PEOPLE HAVE

~~**The time to fill a tank is the volume times the rate.**~~

Why it feels right: When we find a distance we multiply, so multiplying looks like the right thing to do.

The test that breaks it: Volume 120 litres times rate 12 litres per minute is 1 440. That would be a whole day for a small tank. Each minute adds 12 litres, so you count how many 12s fit in 120. That is a division, 10 minutes.

What is true: Time = volume ÷ rate. Multiply only when you want a volume: volume = rate × time.

KEY WORDS

rate of flow
how much liquid moves in each unit of time, such as litres per minute.

gradient
how steep a line is: the change up divided by the change across.

BACK TO YOUR GUESS

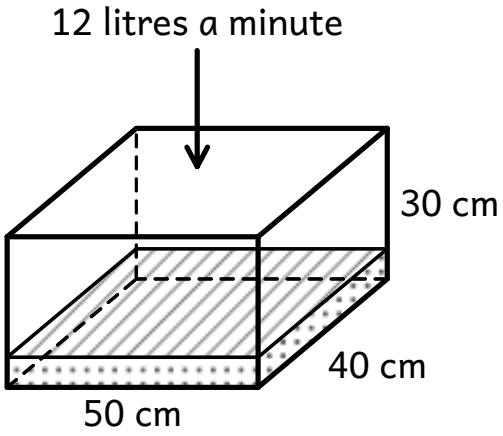
The answer is A: 10 minutes, because 120 ÷ 12 = 10.

Each minute adds 12 litres, so you count how many lots of 12 litres make 120. That is 120 ÷ 12 = 10 minutes. Taking one number from the other has no meaning, and 1 440 minutes would be a whole day.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. A tank is 50 cm long, 40 cm wide and 30 cm deep. It starts empty, and the picture shows it part way through filling. A tap fills it at 12 litres a minute. How long does it take to fill the whole tank?



   A 5 000 minutes

   B 0.2 minutes

   C 5 minutes

   Answer at the back of the book.

2. Water flows from a hose at 8 litres per minute for 15 minutes. How many litres flow?

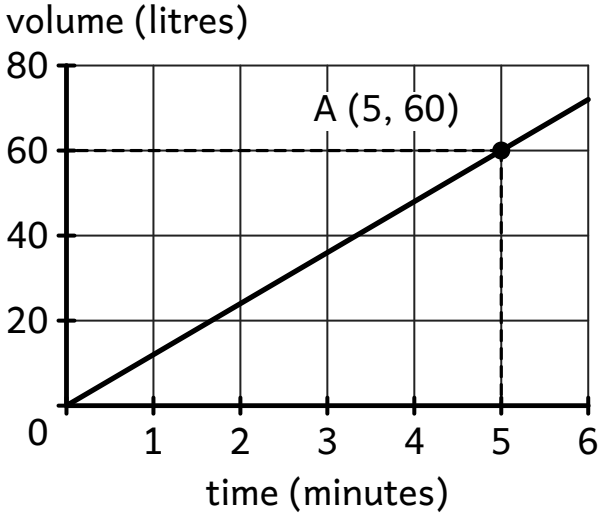
   A 120 litres

   B 1.9 litres

   C 23 litres

   Answer at the back of the book.

3. The graph shows water running steadily into a tank. The line goes through (0, 0) and (5, 60). What is the rate of flow?



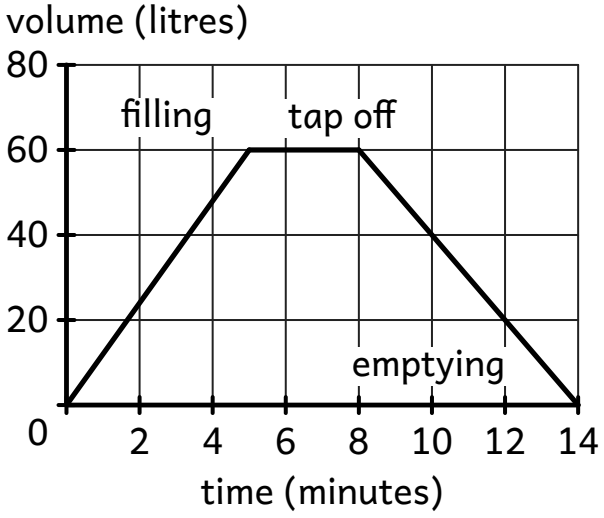
A 60 litres per minute

B 12 litres per minute

C 5 litres per minute

Answer at the back of the book.

4. Look at the graph of Hana's tank, which holds 120 litres. What does the flat part, from 5 to 8 minutes, tell you?



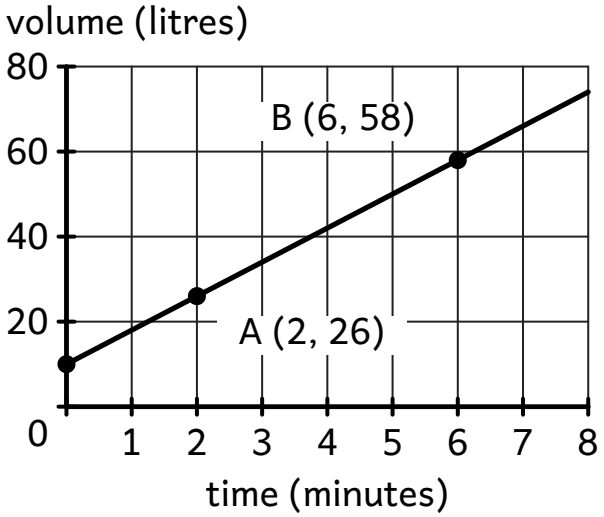
A The tank is full.

B Water flows in at a steady rate.

C No water flows in or out.

Answer at the back of the book.

5. A tank already holds 10 litres. Water then runs in steadily. The graph of volume against time is a straight line through (2, 26) and (6, 58). What is the rate of flow?



A 8 litres per minute

B 9.7 litres per minute

C 32 litres per minute

Answer at the back of the book.

6. **From an earlier chapter**

A block has a mass of 360 g and a volume of 120 cm^3. What is its density, in g/cm^3?

Answer at the back of the book.

7. A tank holds 180 litres. A tap fills it from empty at 15 litres per minute. How many minutes does it take?

Answer at the back of the book.

8. Water flows from a pipe at 18 litres per minute for 25 minutes. How many litres flow?

Answer at the back of the book.

9. A straight line graph of volume against time goes through (2, 14) and (6, 38). What is the rate of flow, in litres per minute?

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Rate = volume ÷ time. Volume = rate × time. Time = volume ÷ rate. On a volume time graph the gradient is the rate. Flat means no flow. Down means water leaves.

Draw: A volume time graph with a rising line, a flat part and a falling line, each part labelled.

Key words, A to Z

Every key word in this book, what it means, and the chapter that teaches it.

apex
the point at the top of a pyramid or a cone. M4.5.2

average speed
the total distance of a journey divided by the total time. M4.5.6

compound unit
a unit made of two units, such as km per hour or metres per second. M4.5.6

cross section
the flat shape you see when you cut a solid straight across. M4.5.1

curved surface
the side of a cylinder, without its two ends. M4.5.1

density
the mass in each unit of volume. Density = mass ÷ volume. M4.5.7

frustum
what is left of a cone when its tip is cut off by a cut parallel to the base. M4.5.4

gradient
how steep a line is: the change up divided by the change across. M4.5.8

half sphere
half of a sphere, cut through its centre, with a flat circle for a face. M4.5.3

litre
a unit for liquids: the volume of a cube 10 cm on each edge, which is 1 000 cm^3. M4.5.5

millilitre
one thousandth of a litre. 1 ml is the same as 1 cm^3. M4.5.5

net
the faces of a solid laid out flat. M4.5.1

pascal
the unit of pressure. One pascal is one newton per m^2. M4.5.7

perpendicular height
the height from the apex straight down to the base, at a right angle to it. M4.5.2

pressure

the force on each unit of area. Pressure = force ÷ area. M4.5.7

prism
a solid whose cross section stays the same all the way along. M4.5.1

rate of flow
how much liquid moves in each unit of time, such as litres per minute. M4.5.8

similar
the same shape but a different size, with all the lengths in the same ratio. M4.5.4

slant height
the length from the apex down the sloping side to the base edge. M4.5.2

sphere
a solid ball shape: every point of its surface is the same distance from the centre. M4.5.3

Answers

M4.5.1. Prisms and cylinders

1. **A tent shaped like a triangle, with the same triangle at both ends**

   The tent has the same triangle all the way along, so it is a triangular prism. The cone and the ball change size as you cut across them.

   - **If you chose A:** A cone gets narrower to a point, so its cross section changes along its height. A prism keeps the same cross section. Go to M4.5.1.
   - **If you chose C:** A cut across a ball gives circles of different sizes. A prism gives the same shape every time. Go to M4.5.1.

2. **360 cm^3**

   The end area is ½ × 15 × 8 = 60 cm^2. The volume is 60 × 6 = 360 cm^3. The 17 cm side is a slanted side and is not needed.

   - **If you chose A:** You multiplied 15 × 8 × 6 and forgot that the end face is a triangle. A triangle is half of the 15 by 8 rectangle. Go to M4.5.1.
   - **If you chose B:** You may have added the three sides of the triangle, 15 + 17 + 8 = 40, and multiplied by 6. The volume needs the area of the end face, not its perimeter. Go to M3.6.4.

3. **785.4 cm^3**

   The end area is π × 5^2 = 25π. The volume is 25π × 10 = 250π, which is 785.4 cm^3.

   - **If you chose B:** You used the diameter, 10 cm, in place of the radius. That makes the answer four times too big. Go to M4.5.1.
   - **If you chose C:** That is π × 5 × 10. The end area is π × 5^2, not π × 5. Go to M4.5.1.
   - **If you chose D:** That is 2π × 5 × 10, the area of the curved surface. The volume is the end area times the height. Go to M4.5.1.

4. **480 cm^3**

   The end area is ½ × (10 + 6) × 5 = 40 cm^2. The volume is 40 × 12 = 480 cm^3.

   - **If you chose A:** You found (10 + 6) × 5 × 12 and left out the ½ in the area of a trapezium. Go to M3.6.5.
   - **If you chose C:** You used only the long parallel side, 10 × 5 × 12. A trapezium has two parallel sides, and the area uses both of them. Go to M3.6.5.

5. **88.0 cm^2**

   The curved surface is $2\pi \times 2 \times 5 = 20\pi$. The two ends are $2 \times \pi \times 2^2 = 8\pi$. The total is 28π, which is 88.0 cm^2, to 1 decimal place.

   - **If you chose A:** That is the curved surface alone, $2\pi \times 2 \times 5$. A closed cylinder also has two circular ends. Go to M4.5.1.
   - **If you chose B:** That is the two circular ends alone, $2 \times \pi \times 2^2$. The curved surface is missing. Go to M4.5.1.
   - **If you chose C:** That is right for a tin with no lid, with only one circular end. This cylinder is closed, so it has two. Go to M4.5.1.

6. **4 times as big**

   The end area is πr^2. Doubling r multiplies it by 2^2, which is 4. The height has not changed, so the volume is 4 times as big.

   - **If you chose A:** The radius doubles, but the radius is squared in πr^2. The area of the end face grows by more than 2 times. Go to M4.5.1.
   - **If you chose B:** A volume grows by 8 times when all three lengths double. Here the height stays the same, so only the end area changes. Go to M4.4.6.

7. **180 cm^3**

   The end area is $\frac{1}{2} \times 8 \times 5 = 20$ cm^2. The volume is $20 \times 9 = 180$ cm^3.

8. **351.9 cm^3**

   The end area is $\pi \times 4^2 = 16\pi$. The volume is $16\pi \times 7 = 112\pi$, which is 351.9 cm^3 to 1 decimal place.

M4.5.2. Pyramids and cones

1. **301.6 cm^3**

   The base area is $\pi \times 6^2 = 36\pi$. The volume is $\frac{1}{3} \times 36\pi \times 8 = 96\pi$, which is 301.6 cm^3 to 1 decimal place.

   - **If you chose A:** You used the slant height, 10 cm, in place of the perpendicular height, 8 cm. The volume uses the height at a right angle to the base. Go to M4.5.2.
   - **If you chose B:** You forgot the $\frac{1}{3}$. $\pi \times 6^2 \times 8$ is the volume of the cylinder round the cone. The cone holds a third of it. Go to M4.5.2.

2. **60 cm³**

   The base area is 5 × 4 = 20 cm². The volume is ⅓ × 20 × 9 = 60 cm³.

   - **If you chose A:** You found base area × height, 20 × 9. That is the volume of the prism with the same base and height. The pyramid is a third of it. Go to M4.5.2.
   - **If you chose C:** You took half of 20 × 9. A pyramid fills one third of the prism, not one half. Go to M4.5.2.

3. **204.2 cm²**

   The curved surface area is π × 5 × 13 = 65π, which is 204.2 cm² to 1 decimal place.

   - **If you chose B:** You used the perpendicular height, 12 cm, in πrh. The curved surface uses the slant height, 13 cm. Go to M4.5.2.
   - **If you chose C:** You used 2πrl, which is twice the right amount. The sector of a cone has area πrl, not 2πrl. Go to M4.5.2.

4. **3 cones**

   The cone holds 30π cm³ and the cylinder holds 90π cm³. 90 ÷ 30 = 3, so the cylinder holds 3 cones of rice.

   - **If you chose A:** You may think a cone is half of its cylinder. Half of 90π is 45π, but the cone holds only 30π, so 2 cones are not enough. Go to M4.5.2.
   - **If you chose B:** The cone has the volume ⅓πr²h, and the cylinder has πr²h. The cylinder is 3 times as big, not 4. Go to M4.5.2.

5. **10 cm**

   $l^2 = 6^2 + 8^2 = 36 + 64 = 100$, so $l = \sqrt{100} = 10$ cm.

   - **If you chose A:** You added 6 and 8. The slant height is the longest side of a right angled triangle, so it comes from $l^2 = r^2 + h^2$. Go to M4.3.2.
   - **If you chose C:** That is l^2, $6^2 + 8^2$. The slant height is the square root of it. Go to M4.3.1.

6. **251.3 cm³**

   The end area is π × 4² = 16π. The volume is 16π × 5 = 80π, which is 251.3 cm³ to 1 decimal place.

7. **108 cm³**

   The base area is 6 × 6 = 36 cm². The volume is ⅓ × 36 × 9 = 108 cm³.

8. **150.8 cm^3**

   The base area is $\pi \times 4^2 = 16\pi$. The volume is $\frac{1}{3} \times 16\pi \times 9 = 48\pi$, which is 150.8 cm^3 to 1 decimal place.

9. **424.1 cm^2**

   The curved surface area is $\pi \times 9 \times 15 = 135\pi$, which is 424.1 cm^2 to 1 decimal place.

M4.5.3. Spheres

1. **523.6 cm^3**

   $V = 4/3 \times \pi \times 5^3 = 4/3 \times \pi \times 125 = 500\pi/3$, which is 523.6 cm^3 to 1 decimal place.

   - **If you chose A:** That is $4 \times \pi \times 5^2$, the formula for the surface area. The volume has r cubed and a 4/3 in front. Go to M4.5.3.
   - **If you chose C:** You left out the $^4/_3$. The volume of a sphere is $^4/_3 \times \pi \times 5^3$, not $\pi \times 5^3$. Go to M4.5.3.

2. **201.1 cm^2**

   $A = 4 \times \pi \times 4^2 = 4 \times \pi \times 16 = 64\pi$, which is 201.1 cm^2 to 1 decimal place.

   - **If you chose A:** That number comes from the volume formula, $4/3 \times \pi \times 4^3$. The surface area is $4 \times \pi \times 4^2$, and it is measured in cm^2. Go to M4.5.3.
   - **If you chose B:** You used the diameter, 8 cm, in place of the radius, 4 cm. Go to M4.5.3.

3. **56.5 cm^3**

   The whole sphere has $V = 4/3 \times \pi \times 27 = 36\pi$. The half sphere has $36\pi \div 2 = 18\pi$, which is 56.5 cm^3 to 1 decimal place.

   - **If you chose B:** That is the volume of the whole sphere. The half sphere has half of it. Go to M4.5.3.
   - **If you chose C:** You halved the radius, 1.5 cm, instead of halving the volume. A half sphere has the same radius as the whole sphere. Go to M4.5.3.

4. **235.6 cm^2**

   The curved part is $2\pi \times 5^2 = 50\pi$. The flat face is $\pi \times 5^2 = 25\pi$. The total is 75π, which is 235.6 cm^2 to 1 decimal place.

   - **If you chose A:** That is the curved part alone, $2\pi \times 5^2$. The question says to count the flat face too. Go to M4.5.3.
   - **If you chose B:** That is $4\pi \times 5^2$, the surface of the whole sphere. The half sphere has half of that, plus the flat face. Go to M4.5.3.

5. **8 times as big**

   The volume goes with r^3. The radius is multiplied by 2, so the volume is multiplied by $2^3 = 8$.

   - **If you chose B:** The radius is twice as big, but the volume has r^3 in it. A sphere that is twice as wide is much more than twice as big. Go to M4.5.3.
   - **If you chose C:** That is how many times the surface area grows. The volume goes with r^3, not r^2. Go to M4.4.6.

6. **235.6 cm^3**

   The base area is $\pi \times 5^2 = 25\pi$. The volume is $\frac{1}{3} \times 25\pi \times 9 = 75\pi$, which is 235.6 cm^3 to 1 decimal place.

7. **3 053.6 cm^3**

   $V = 4/3 \times \pi \times 9^3 = 4/3 \times \pi \times 729 = 972\pi$, which is 3 053.6 cm^3 to 1 decimal place.

8. **615.8 cm^2**

   $A = 4 \times \pi \times 7^2 = 4 \times \pi \times 49 = 196\pi$, which is 615.8 cm^2 to 1 decimal place.

9. **134.0 cm^3**

   The whole sphere has volume $4/3 \times \pi \times 64$, which is 268.08 cm^3 to 2 decimal places. Half of that is 134.04, which is 134.0 cm^3 to 1 decimal place.

M4.5.4. Joined and cut solids

1. **603.2 cm³**

   The cylinder is $\pi \times 4^2 \times 10 = 160\pi$. The cone is $\frac{1}{3} \times \pi \times 4^2 \times 6 = 32\pi$. The total is 192π, which is 603.2 cm³ to 1 decimal place.

   - **If you chose B:** You treated the whole solid as one cylinder of height 16 cm. The cone is only a third of a cylinder of its own height. Go to M4.5.4.
   - **If you chose C:** That is the cylinder alone, $\pi \times 4^2 \times 10$. You also need the cone on top. Go to M4.5.4.

2. **586.4 cm³**

   The big cone has height 10, so $\frac{1}{3} \times \pi \times 8^2 \times 10 = 670.2$. The tip is $\frac{1}{3} \times \pi \times 4^2 \times 5 = 83.8$. The pot is $670.2 - 83.8 = 586.4$ cm³, to 1 decimal place.

   - **If you chose A:** You used a cylinder with the average radius, 6 cm. A frustum is not a cylinder, so this is only close. Go to M4.5.4.
   - **If you chose B:** That is the whole big cone, with the tip still in it. The tip was cut away, so subtract its volume. Go to M4.5.4.

3. **4 cm**

   Let the tip have height x. Then $x \div (x + 8) = 2 \div 6$. So $6x = 2x + 16$, $4x = 16$, and $x = 4$ cm.

   - **If you chose A:** You multiplied the height of the pot, 8 cm, by $2 \div 6$. The ratio $2 \div 6$ compares the tip with the whole cone, not with the pot. Go to M4.5.4.
   - **If you chose C:** That is the height of the whole cone. The tip is only the top part, which is shorter. Go to M4.5.4.

4. **197.9 cm³**

   The cylinder is $\pi \times 3^2 \times 5 = 45\pi$. The cone is $\frac{1}{3} \times \pi \times 3^2 \times 6 = 18\pi$. The total is 63π, which is 197.9 cm³ to 1 decimal place.

   - **If you chose B:** You treated the cone as a cylinder, with no ⅓. $\pi \times 3^2 \times 11 = 311.0$ is for one tube 11 cm high. Go to M4.5.2.
   - **If you chose C:** That is the cylinder alone, $\pi \times 3^2 \times 5$. The cone on top has volume too. Go to M4.5.4.

5. **$^1/_8$**

   Every length is halved, so the volume is multiplied by ½ × ½ × ½ = 1/8. The frustum is the other 7/8.

   - **If you chose A:** The tip is half as tall, but it is also half as wide. A volume has three lengths in it, so the tip is much less than half. Go to M4.4.6.
   - **If you chose C:** A factor of 2 × 2 = 4 is for areas. A volume has three lengths in it, so you multiply three times. Go to M4.4.6.

6. **904.8 cm³**

   V = 4/3 × π × 6^3 = 4/3 × π × 216 = 288π, which is 904.8 cm^3 to 1 decimal place.

7. **502.7 cm³**

   The cylinder is π × 4^2 × 7 = 112π. The cone is ⅓ × π × 4^2 × 9 = 48π. The total is 160π, which is 502.7 cm^3 to 1 decimal place.

8. **329.9 cm³**

   The tip x has x ÷ (x + 5) = 3 ÷ 6, so x = 5. The whole cone is ⅓ × π × 6^2 × 10 = 120π. The tip is ⅓ × π × 3^2 × 5 = 15π. The pot is 105π, which is 329.9 cm^3 to 1 decimal place.

M4.5.5. Changing units of area and volume

1. **25 000 cm²**

   One square metre is 10 000 cm^2. So 2.5 m^2 is 2.5 × 10 000 = 25 000 cm^2.

   - **If you chose B:** You multiplied by 100, the number of cm in a metre. For an area you multiply by 100 × 100. Go to M4.5.5.
   - **If you chose C:** You multiplied by 1 000. A square metre has 100 cm along each edge, so the number is 100 × 100, which is 10 000. Go to M4.5.5.

2. **3.6 m²**

   One square metre is 10 000 cm^2. So 36 000 cm^2 is 36 000 ÷ 10 000 = 3.6 m^2.

   - **If you chose A:** You divided by 100. To change cm^2 into m^2 you divide by 10 000, because 1 m^2 = 10 000 cm^2. Go to M4.5.5.
   - **If you chose C:** You divided by 1 000. To change cm^2 into m^2 you divide by 10 000. Go to M4.5.5.

3. **60 litres**

   The volume is 50 × 40 × 30 = 60 000 cm^3. One litre is 1 000 cm^3, so the tank holds 60 000 ÷ 1 000 = 60 litres.

   - **If you chose A:** That is the volume in cm^3. A litre is 1 000 cm^3, so divide by 1 000 to get the number of litres. Go to M4.5.5.
   - **If you chose B:** You divided by 100. One litre is 1 000 cm^3, not 100 cm^3. Go to M4.5.5.

4. **4 000 000 cm^3**

   One cubic metre is 1 000 000 cm^3. So 4 m^3 is 4 × 1 000 000 = 4 000 000 cm^3.

   - **If you chose A:** You multiplied by 100 000. One cubic metre is 100 × 100 × 100 cm^3, which is 1 000 000, so there is one more zero. Go to M4.5.5.
   - **If you chose C:** That is the number of litres. The question asks for cm^3, and each litre is 1 000 cm^3. Go to M4.5.5.

5. **2.4 m^3**

   One cubic metre is 1 000 litres. So 2 400 litres is 2 400 ÷ 1 000 = 2.4 m^3.

   - **If you chose B:** You divided by 100. One m^3 is 1 000 litres, so divide by 1 000. Go to M4.5.5.
   - **If you chose C:** You divided by 10. One m^3 holds 1 000 litres, so 2 400 litres is a little over 2 m^3. Go to M4.5.5.

6. **169.6 cm^3**

   The cylinder is $\pi \times 3^2 \times 4 = 36\pi$. The cone is $\frac{1}{3} \times \pi \times 3^2 \times 6 = 18\pi$. The total is 54π, which is 169.6 cm^3 to 1 decimal place.

7. **8 000 cm^2**

   One square metre is 10 000 cm^2. So 0.8 m^2 is 0.8 × 10 000 = 8 000 cm^2.

8. **90 litres**

   The volume is 60 × 30 × 50 = 90 000 cm^3. One litre is 1 000 cm^3, so the tank holds 90 litres.

9. **3.5 m^3**

   One cubic metre is 1 000 000 cm^3. So 3 500 000 cm^3 is 3 500 000 ÷ 1 000 000 = 3.5 m^3.

M4.5.6. Speed, distance and time

1. **60 km/h**

   Speed = distance ÷ time = 150 ÷ 2.5 = 60 km/h.

   - **If you chose B:** You multiplied the distance by the time. Speed is the distance divided by the time. Go to M4.5.6.
   - **If you chose C:** You added the numbers. Speed is how far you go in each hour, so divide 150 by 2.5. Go to M4.5.6.

2. **30 km**

   40 minutes is 40 ÷ 60 = 2/3 of an hour. Distance = speed × time = 45 × 2/3 = 30 km.

   - **If you chose A:** You multiplied 45 by 40 and left the time in minutes. The speed is per hour, so change 40 minutes into hours first. Go to M4.5.6.
   - **If you chose B:** You divided the speed by the time. To find the distance, multiply speed by time, with the time in hours. Go to M4.5.6.

3. **3.5 hours**

   Time = distance ÷ speed = 210 ÷ 60 = 3.5 hours, which is 3 hours 30 minutes.

   - **If you chose A:** You read 3.5 hours as 3 hours 50 minutes. The 0.5 is half an hour, which is 30 minutes. Go to M4.5.6.
   - **If you chose B:** You divided the speed by the distance. Time is the distance divided by the speed. Go to M4.5.6.
   - **If you chose C:** You multiplied the two numbers. Time = distance ÷ speed. Go to M4.5.6.

4. **25 m/s**

   To change km/h into m/s, divide by 3.6. 90 ÷ 3.6 = 25 m/s.

   - **If you chose B:** You multiplied by 3.6. That changes m/s into km/h. To change km/h into m/s, divide by 3.6. Go to M4.5.6.
   - **If you chose C:** You divided by 60, which changes km per hour into km per minute. For m/s, divide by 3.6. Go to M4.5.6.

5. **40 km/h**

   The first part takes 30 ÷ 60 = 0.5 hours. The second part takes 30 ÷ 30 = 1 hour. The total is 60 km in 1.5 hours, so the average speed is 60 ÷ 1.5 = 40 km/h.

   - **If you chose A:** You found the mean of 60 and 30. The car spends longer at 30 km/h, so the average is lower than 45. Go to M4.5.6.
   - **If you chose B:** That is the first speed. The average uses the total distance and the total time of both parts. Go to M4.5.6.

6. **42 000 cm^2**

   One square metre is 10 000 cm^2. So 4.2 m^2 is 4.2 × 10 000 = 42 000 cm^2.

7. **24 km/h**

   45 minutes is 45 ÷ 60 = 0.75 hours. Speed = 18 ÷ 0.75 = 24 km/h.

8. **15 m/s**

   To change km/h into m/s, divide by 3.6. 54 ÷ 3.6 = 15 m/s.

9. **46.7 km/h**

   The first part takes 80 ÷ 40 = 2 hours. The second part takes 60 ÷ 60 = 1 hour. The total is 140 km in 3 hours, so the average speed is 140 ÷ 3 = 46.7 km/h to 1 decimal place.

M4.5.7. Density and pressure

1. **2.5 g/cm^3**

   Density = mass ÷ volume = 150 ÷ 60 = 2.5 g/cm^3.

   - **If you chose A:** You multiplied the mass by the volume. Density is the mass shared out over the volume, so divide. Go to M4.5.7.
   - **If you chose C:** You divided the volume by the mass. Density = mass ÷ volume, with the mass on the top. Go to M4.5.7.

2. **1 080 g**

   The volume is 10 × 8 × 5 = 400 cm^3. Mass = density × volume = 2.7 × 400 = 1 080 g.

   - **If you chose B:** You divided the volume, 400 cm^3, by the density. To find the mass, multiply the density by the volume. Go to M4.5.7.
   - **If you chose C:** You used the area of one face, 10 × 8 = 80 cm^2, in place of the volume. The mass needs the whole volume, 10 × 8 × 5 = 400 cm^3. Go to M4.5.7.

3. **60 cm^3**

   Volume = mass ÷ density = 480 ÷ 8 = 60 cm^3.

   - **If you chose A:** You multiplied the mass by the density. To find the volume, divide the mass by the density. Go to M4.5.7.
   - **If you chose B:** You divided the density by the mass. The volume is the mass divided by the density. Go to M4.5.7.

4. **12 000 Pa**

   Pressure = force ÷ area = 600 ÷ 0.05 = 12 000 N/m^2, which is 12 000 Pa.

   - **If you chose A:** You multiplied the force by the area. Pressure = force ÷ area. Go to M4.5.7.
   - **If you chose C:** You divided the area by the force. Pressure is the force on each m^2, so the force is on the top. Go to M4.5.7.

5. **About 0.6 N/cm^2**

   The area is 5 × 2 = 10 cm^2. The pressure is about 6 ÷ 10 = 0.6 N/cm^2.

   - **If you chose A:** You multiplied 6 by 10. Pressure is the force divided by the area. Go to M4.5.7.
   - **If you chose B:** You used a large face, 8 cm by 5 cm, which is 40 cm^2. The block stands on the small face, 5 cm by 2 cm. It has less area and gives a bigger pressure. Go to M4.5.7.

6. **35 m/s**

   To change km/h into m/s, divide by 3.6. 126 ÷ 3.6 = 35 m/s.

7. **3 g/cm^3**

   Density = mass ÷ volume = 480 ÷ 160 = 3 g/cm^3.

8. **300 g**

   The volume is 4 × 5 × 6 = 120 cm^3. Mass = density × volume = 2.5 × 120 = 300 g.

9. **3 N/cm^2**

   Pressure = force ÷ area = 90 ÷ 30 = 3 N/cm^2.

M4.5.8. Rates of flow and rate graphs

1. **5 minutes**

   The volume is 50 × 40 × 30 = 60 000 cm^3, which is 60 litres. Time = 60 ÷ 12 = 5 minutes.

   - **If you chose A:** You divided the volume in cm^3, 60 000, by 12. The rate is in litres, so change the volume into litres first, 60 000 ÷ 1 000 = 60. Go to M4.5.5.
   - **If you chose B:** You divided the rate by the volume. Time = volume ÷ rate, with the volume on the top. Go to M4.5.8.

2. **120 litres**

   Volume = rate × time = 8 × 15 = 120 litres.

   - **If you chose B:** You divided the time by the rate, 15 ÷ 8. To find a volume, multiply the rate by the time. Go to M4.5.8.
   - **If you chose C:** You added the two numbers. In each of 15 minutes the hose gives 8 litres, so multiply 8 by 15. Go to M4.5.8.

3. **12 litres per minute**

   The line starts at (0, 0), so the gradient is 60 ÷ 5 = 12 litres per minute.

   - **If you chose A:** You read the height of point A. That is the volume after 5 minutes. The rate is the volume divided by the time. Go to M4.5.8.
   - **If you chose C:** You read the time of point A. The rate is the volume divided by the time, 60 ÷ 5. Go to M4.5.8.

4. **No water flows in or out.**

   The gradient of the flat part is 0. The volume stays at 60 litres, so no water goes in and none goes out.

   - **If you chose A:** The tank holds 120 litres, and the flat part is at 60 litres. A flat part shows that the volume is not changing, not that the tank is full. Go to M4.5.8.
   - **If you chose B:** If water flowed in, the line would rise. The line is flat, so the volume stays the same and the rate is 0. Go to M4.5.8.

5. **8 litres per minute**

   The volume goes up by 58 − 26 = 32 litres while the time goes up by 6 − 2 = 4 minutes. The gradient is 32 ÷ 4 = 8 litres per minute.

   - **If you chose B:** You divided the volume at B, 58, by its time, 6. The line does not start at 0 litres, so use the change in volume and the change in time. Go to M4.5.8.
   - **If you chose C:** You found the change in volume, 58 − 26 = 32. That is the rise. Divide it by the change in time, 6 − 2 = 4. Go to M4.5.8.

6. **3 g/cm³**

   Density = mass ÷ volume = 360 ÷ 120 = 3 g/cm^3.

7. **12 minutes**

   Time = volume ÷ rate = 180 ÷ 15 = 12 minutes.

8. **450 litres**

   Volume = rate × time = 18 × 25 = 450 litres.

9. **6 litres per minute**

   The volume goes up by 38 − 14 = 24 litres while the time goes up by 6 − 2 = 4 minutes. The gradient is 24 ÷ 4 = 6 litres per minute.

About this book

For learners who can find the area of a circle and the volume of a box. In many schools this fits teenagers of about 14 to 16, and adults who want to refresh area, volume, speed and density can start here too. Each chapter takes one or two sessions. A learner can work alone: every exercise has its answer and the reason for it.

Written and drawn by attnom. Edition 1.0, 2026-11-12.

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